# TIMETABLE and ABSTRACTS

## Logic Colloquium 2004

**25-31 July**
**Torino Italy**

# Contents

# Week overview

| | Mon 26 | Tue 27 | Wed 28 | Thu 29 | Fri 30 | Sat 31 |
|---|---|---|---|---|---|---|
| 9:00–10:00 | Opening | Willard (tutorial) | Willard (tutorial) | Willard (tutorial) | Tamburrini | Valeriote |
| 10:00–11:00 | Mints | Scanlon (tutorial) | Scanlon (tutorial) | Bartoszynski | Scanlon (tutorial) | Berarducci |
| 11:20–12:20 | Solomon (tutorial) | Solomon (tutorial) | Kechris Solecki (tutorial) | Solomon (tutorial) | Kechris Solecki (tutorial) | Kechris Solecki (tutorial) |
| 12:20 | | | | Excursion | | |
| 14:00 | | | Contributed | | | |
| 14:20–15:20 | Marcone | Special Sessions | Contributed<br>Contributed<br>Contributed | | Special Sessions | Baldwin |
| 15:20–16:20 | Stephan | Special Sessions | Contributed<br>Contributed<br>Contributed | | Special Sessions | Woodin |
| 16:40–17:40 | Möllerfeld | Contributed<br>Contributed<br>Contributed | Contributed<br>Contributed<br>Contributed | | Contributed<br>Contributed<br>Contributed | |
| 17:40–18:40 | Sundholm | Contributed<br>Contributed<br>Contributed | Contributed<br>Contributed<br>Contributed | | 18:10 Dinner | |
| 18:40–19:40 | | Contributed<br>Contributed<br>Contributed | Contributed<br>Contributed<br>Contributed | | | |
| 20:10 | Reception | Panel Discussion (20:10–22:00) | | | | |

# Monday 26

| Time | Sala G.Agnelli |
|---|---|
| 9:00 – 10:00 | **Opening** |
| 10:00 – 11:00 | **Mints**, ***Intuitionistic Frege systems are polynomially equivalent*** |
| 11:20 – 12:20 | **Solomon**, ***Computable algebra I*** |
| 14:20 – 15:20 | **Marcone**, ***Logical aspects of wqo theory*** |
| 15:20 – 16:20 | **Stephan**, ***Kolmogorov complexity*** |
| 16:40 – 17:40 | **Möllerfeld**, ***On the proof theory of*** $\Pi^1_2$***-comprehension*** |
| 17:40 – 18:40 | **Sundholm**, ***The META linguistic turn*** |

**Welcome reception in the garden 20:00 – 22:30**

# Tuesday 27

| | Sala G.Agnelli |
|---|---|
| 9:00 – 10:00 | **Willard,** *An overview of modern universal algebra I* |
| 10:00 – 11:00 | **Scanlon,** *Geometric stability theory I* |
| 11:20 – 12:20 | **Solomon,** *Computable algebra II* |

**Panel Discussion 20:10 – 22:00 Sala 200**

***Kant's legacy for the philosophy of logic***

**Mirella Capozzi**
**Cesare Cozzo**
**Riccardo Pozzo**
**Dag Prawitz (moderator)**

| | Sala 200 | Sala Torino | Sala G.Agnelli |
|---|---|---|---|
| 14:20 | **Tomašić,** *Probabilistic independence theorem in pseudofinite fields* | **Aglianò,** *Basic-algebras: an overview* | **Setzer** *Universes in type theory: Mahlo and $\Pi_3$-reflection* |
| 14:50 | **Vidaux,** *The analogue of Büchi's problem in polynomial rings* | **Vertési,** *Checking equations in finite algebras* | |
| 15:20 | **Berenstein,** *Probability spaces with a generic automorphism* | **Givant,** *Interconnections between groups and algebras of relations* | **Cantini** *Classifications, operations and (constructive) sets* |
| 15:50 – 16:20 | **Servi,** *O-minimal exponential fields* | **Larose,** *On the complexity of strong colourings of hypergraphs* | |

| | Sala G.Agnelli | Sala 200 | Sala Torino | Sala Piramide | Sala del Consiglio |
|---|---|---|---|---|---|
| 16:40 | **Semenova** *CANCELED* | **Hirschorn** *On iterated (proper) forcing without adding reals* | **Berardi** *Classical arithmetic as solution of a maximum problem* | **Dobrinen** *Almost everywhere domination* | **Ehrlich** *... Non-archimedean mathematics and the roots of a misconception* |
| 17:00 | **Barbina** *Reconstruction of classical geometries from their automorphism group* | **Kanovei** *Fully saturated extensions of standard universe* | **Robles** *Converse Ackermann property and constructive negation ...* | **Giorgi** *A high noncuppable enumeration degree* | **Bruni** *A note on Gödel's philosophy of mathematics in the light of his* Nachlass |
| 17:20 | **Darniere** *Model completion of scaled lattices* | **Di Nasso** *Different paths to nonstandard analysis* | **Schreiner** *Automatical recognition of the pretabularity and ...* | **Podzorov** *Rogers semilattices of arithmetical numberings* | **Meyer** *Boole bites back* |
| 17:40 | **Guzy** *Topological differential structures* | **Kurilić** *Ultrafilters in models of set theory* | **Chubaryan** *On the proofs complexity in the resolution systems of intuitionistic ...* | **Puzarenko** *On computability in structures* | **Akıncı** *David Lewis's counterpart theory* |
| 18:00 | **Kontinen** *On definability of second order generalized quantifiers* | **Yorioka** *Gap spectra under Martin's axiom* | **Kretz** *Deduction chains for logic of common knowledge* | **Stukachev** *On structures decidable in admissible sets* | **da Silva** *On the principle of excluded middle* |
| 18:25 | **Belegradek** *Collapse under total independence* | **Bosch** *Definably large cardinals* | **Mostowski** *Arithmetic of coprimality in finite models* | **Frolov** *Computable copies of distributive lattices with relative complements ...* | **Ronzitti** *On the philosophy of intuitionistic mathematics* |
| 18:45 | **Engström** *Notions of resplendency for logics stronger than 1st-order logic* | **Vuksanovic** *A canonical partition theorem for trees* | **Paśniczek** *CANCELED* ⇑ **Anticipated** | **Alaev** *Strongly computable Boolean algebras* | **Faas** *Visual and heterogeneous inferences* |
| 19:05 | **Krupiński** *A special thin type* | **Koenig** *Partial orderings with dense subtrees* | **Libert** *Topological solutions to the Fregean problem and their corresponding AST* | **Kogabaev** *Computable lists of I-algebras and complexity of some problems* | **Urtubey** *Vagueness, logic and information processing* |
| 19:25 – 19:45 | **Mudrinski** *... Endomorphism monoids of countable homogeneous graphs* | **Irrgang** *Higher-gap morasses and morass constructions* | | **Henk** *A qualitative approach to the Church-Turing thesis* | **Pallares Vega** *Model theory and philosophy of language* |

# Wednesday 28

| Time | Sala G.Agnelli |
|---|---|
| 9:00–10:00 | Willard, *An overview of modern universal algebra II* |
| 10:00–11:00 | Scanlon, *Geometric stability theory II* |
| 11:20–12:20 | Kechris and Solecki, *Logic and dynamical systems I* |

| Time | Sala G.Agnelli | Sala 200 | Sala Torino | Sala Piramide | Sala del Consiglio |
|---|---|---|---|---|---|
| 14:00 | **Kysiak** *Nonmeasurable algebraic sums of sets of reals* | | **Hosni** *Rationality as conformity* | **Galatos** *Translations in substructural logic* | **Tulenheimo** *Independence-friendly modal logic* |
| 14:20 | **Kraszewski** *On some properties of ideals on the Cantor space* | **von Plato** *Consistency of arithmetics through sequent calculus in ND style* | **Monteleone** *Thoughts in the age of computer science* | **Weiermann** *Does (concrete) mathematics matter to Gödel's theorem?* | **Naumov** *On modal logic of deductive closure* |
| 14:40 | **Marco** *Invariant model theory and automorphism group actions* | **Zimmermann** *Natural deduction with single assumptions* | **Surowik** ... *Rejection of the thesis of determinism in temporal logic systems* | **Lee** *Kanamori-McAloon principle and the fast growing Ramsey functions* | **Gencer** *Unification and passive inference rules for the modal logic K4* |
| 15:00 | **Pauna** *On pressing down game and Banach–Mazur game* | **Tesconi** *A SN theorem for natural deduction with general elimination rules* | **Arrigoni** *What's the problem with V?* | **Bovykin** *Several proofs of $PA$-unprovability* | **Gheerbrant** *Recursive complexity of L-modal logic* |
| 15:20 | **Zoli** *On Schmidt's $\sigma$-ideal* | **Andou** *A representation of essential reductions in sequent calculus* | **Hauser** *CANCELED* | **Wilken** *Characterizing closure properties of ordinals* | **Negri** *A uniform CE theorem for contraction-free systems of modal logic* |
| 15:40 | **Bold** *AD and Descriptions for cardinals below $\Theta$* | **Font** *CANCELED* | **Bellotti** *The problem of consistency: ZF and large cardinals* | **Heinatsch** *The determinacy strength of $\Pi^1_2$ comprehension* | **Rybakov** *First-order theories of free temporal algebras* |
| 16:00 | **Pinsker** *Set theory in infinite clone theory* | **Brage** *A natural interpretation of classical proofs* | **Zach** *Gödel's 1st incompl. th.m and Detlefsen's Hilbertian instrumentalism* | **Duchhardt** *Reflection vs thinning* | **Lenzi** *Axiomatizing bisimulation quantifiers* |
| 16:20 | | | | | |

| Time | Sala G.Agnelli | Sala 200 | Sala Torino | Sala Piramide | Sala del Consiglio |
|---|---|---|---|---|---|
| 16:40 | **Walczak-Typke** *Algebraic structures in set theory without AC* | **Kun** *On the fundamental group of finite posets* | **Amirzadeh** *Process capability in fuzzy quality control* | **Arai** *Proofs of the termination of rewrite rules for polytime functions* | **Crosilla** *On constructing completions* |
| 17:00 | **Usvyatsov** *Categoricity in complete metric spaces* | **Horváth** *Identities of finite groups* | **Millington** ... *Foundations in a $\lambda$ calculus extension of Lukasiewicz logic* | **Moser** *A note on Cichon's principle* | **Thiel** *Mahlo & Veblen* |
| 17:20 | **Toffalori** *Superdecomposable pure injective modules over integral group rings* | **Grech** ... *The lattice of equational theories of commutative semigroups* | **Buşneag** *Localization for some algebras of fuzzy logic* | **Beltiukov** *Polynomial time programming language* | **Berger** *Unique existence theorems in constructive analysis* |
| 17:40 | **L'Innocente** *Minimalities and modules over rings related to Dedekind domains* | **Markovic** *Star-linear varieties of groupoids* | **Běhounek** *Extensional set equality over Gödel logic* | **Hoogewijs** *Formath: higher-order logic revised* | **Maietti** *Relative topology in constructive mathematics* |
| 18:00 | **Rivière** *Dimension function in the theory of closed ordered differential fields* | **Tasić** *$HSP \neq SHPS$ for commutative rings with identity* | **Horčík** *Strong standard completeness of $\Pi MTL$* | **Gentilini** *Critical deduction chains and inferential models for type theory* | **Bilkova** *A computational view of intuitionistic propositional proofs* |
| 18:25 | **Pierce** *Fields of any characteristic with operators* | **Venema** *McNeille completions of lattice expansions* | **Cintula** *Towards universal fuzzy logic* | **Kullmann** *The combinatorics of negation patterns* | **Aghaei** *A lambda calculus for basic propositional logic* |
| 18:45 | **Petrykowski** *Coverings of groups* | **Ploščica** *Uniform refinement properties in distributive semilattices* | **Jerabek** *Bounded arithmetic in 3-valued logic* | **Rodenburg** *Partial operations and the specification of the stack* | **Ardeshir** *Basic arithmetic* |
| 19:05 | **Majcher** *Absolutely ubiquitous groups* | **Mirek** *Local Finiteness and substructural algebras* | **Maleki** *An algorithm for solving multiobjective linear programming* | **Abdallah** *An algebraic approach to commonsense reasoning* | **Schuster** *The projective spectrum as a distributive lattice* |
| 19:25 | **Khisamiev** *CANCELED* | **McKenzie** *Finite decidability and generative complexity in eq.al cl.es* | | **Skvortsov** ... *Quantified Dummett's logic complete w.r.t. Kripke sheaves* | |
| 19:45 | | | | | |

Sala G.Agnelli | Sala 200 | Sala Torino | Sala Piramide | Sala del Consiglio

# Thursday 29

**Sala G.Agnelli**

| | |
|---|---|
| **9:00** | **Willard**, ***An overview of modern universal algebra III*** |
| **10:00** | **Bartoszynski**, ***Measure and category from set-theoretic viewpoint*** |
| **11:00** | |
| **11:20** | **Solomon**, ***Computable algebra III*** |
| **12:20** | |

**Excursion**

**The bus to the Sacra di San Michele and Abbey of Sant'Antonio di Ranverso leaves at 14:00 from the conference site.**

# Friday 30

| | Sala G.Agnelli |
|---|---|
| 9:00–10:00 | **Tamburrini,** ***Computation and the explanation of intelligent behaviours: ethologically motivated restart*** |
| 10:00–11:00 | **Scanlon,** ***Geometric stability theory III*** |
| 11:20–12:20 | **Kechris and Solecki,** ***Logic and dynamical systems II*** |

| | Sala 200 | Sala Torino | Sala G.Agnelli |
|---|---|---|---|
| 14:20–14:50 | **Cluckers,** *Ax-Kochen-Eršov principles for integrals over local fields* | **Lipparini,** *Tolerance intersection properties* | **Woods** *Probability limit laws for ordinal numbers* |
| 14:50–15:20 | **VanDieren,** *Categoricity for tame abstract elementary classes* | **Aichinger,** *The variety of near-rings is generated by its finite members* | |
| 15:20–15:50 | **Zahidi,** *Elimination theory for addition and Frobenius in rings* | **Zádori,** *Bounded width problems and algebras* | **Lubarsky** *On the independence of variants of power set under IZF–power set* |
| 15:50–16:20 | **Aschenbrenner,** *Faithfully flat Lefschetz extensions* | **Maróti,** *Finite basis problems and results for quasivarieties* | |

| | Sala G.Agnelli | Sala 200 | Sala Torino | Sala Piramide | Sala del Consiglio |
|---|---|---|---|---|---|
| 16:40–17:00 | **D'Aquino** *Quadratic reciprocity law in weak fragments of PA* | **Kozik** *A finite algebra with PSPACE-hard membership problem ...* | **Costantini** *Consistent examples about extensions of continuous functions* | **Parlamento** *Truth in V for $\exists^*\forall\forall$-sentences is decidable* | **Arabpour** *Fuzzy linear regression model with numerical input and fuzzy output* |
| 17:00–17:20 | **Wencel** *A genaralization of o-minimality for expansions of Boolean algebras* | **Delić** *Finite decidability of locally finite varieties* | **Panti** *Substitutions as dynamical systems* | **Kisielewicz** *A new solution to old paradoxes* | **Nehi** *A canonical representation for the solution of fuzzy linear systems ...* |
| 17:20–17:40 | **Leonesi** *On The boolean algebras of definable sets in weakly o-minimal theories* | **Nurakunov** *On lattices of varieties of algebras* | **Lyaletsky** *On types of continuity defined by partially ordered sets ...* | **Denisov** ***CANCELED*** | **Baaz** *Axiomatizability of first-order Gödel logics* |

**Social dinner: The bus to the restaurant leaves at 18:10 (sharp). Dinner starts at 20:00**

# Saturday 31

**Sala G.Agnelli**

| Time | Sala G.Agnelli |
|---|---|
| 9:00–10:00 | **Valeriote**, *The constraint satisfaction problem and universal algebra* |
| 10:00–11:00 | **Berarducci**, *Definable groups in o-minimal structures* |
| 11:20–12:20 | **Kechris and Solecki**, *Logic and dynamical systems III* |
| 14:20–15:20 | **Baldwin**, *Quasiminimal sets and categoricity in abstract elementary classes* |
| 15:20–16:20 | **Woodin**, *Extender models and the inner model problem for supercompact cardinals and beyond* |

**Closing**

# Tutorials

**Reed Solomon,** ***Computable algebra***

**Thomas Scanlon,** ***Geometric stability theory***

**Ross Willard,** ***An overview of modern universal algebra***

**Alexander Kechris & Sławomir Solecki,** ***Logic and dynamical systems***

Reed Solomon

## Computable algebra

This tutorial will focus on a number of themes in computable algebra including computable dimension and categoricity, degree spectra of structures and relations, degrees of isomorphism types, and syntactic characterizations of these notions. The goal is to illustrate these concepts using examples from a wide variety of algebraic structures such as groups, linear orders, Boolean algebras and trees. In addition, we will discuss which classes of algebraic structures display the most general computational behavior found in computable model theory and which classes of structures have algebraic properties which limit their computational behavior. No previous background in computable algebra will be assumed.

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THOMAS SCANLON

## Geometric stability theory

Stability theory was developed to address questions of classification theory in first-order logic and debuted with Morley's celebrated proof of Łoś' conjecture that a countable first-order theory which is categorical in one uncountable cardinal is so in all uncountable cardinals. Techniques from combinatorial geometry were imported into the study of stable theories and proved their worth initially with Zilber's proof of the non-finite axiomatizability of totally categorical theories. A decade ago, Hrushovski applied deep results in geometric stability theory to problems in diophantine geometry. In recent years, it has become clear that in several traditionally geometric theories, such as those of compact complex manifolds and of differential fields, the connections between the combinatorial geometry of geometric stability theory and analytic or differential-algebraic geometry are far tighter than mere analogies or isolated applications. In this tutorial, we trace this history of geometric stability theory paying special attention to its key notions (such as forking, regular types, orthogonality, binding groups, local modularity, *etc*) in geometric theories of independent mathematical interest.

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## Ross Willard

### An overview of modern universal algebra

This tutorial is intended to introduce nonspecialists to the fact that there are deep results in contemporary universal algebra. In the first lecture I will define the context in which universal algebra theory has something to say, and describe some of the basic results upon which much of the work in the field is built. The second lecture will cover the highlights of tame congruence theory, a sophisticated point of view from which to analyze locally finite algebras. The final lecture will describe some of the field's "big" results and open problems concerning finite algebras, notably the undecidability of certain finite axiomatizability problems and related problems, and the so-called "RS problem," currently the most important open problem in the field.

Pure Mathematics Department, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada

Alexander Kechris & Sławomir Solecki

**Logic and dynamical systems**

The main theme of the tutorial will be the interaction between the study of Polish group actions and their orbit spaces with various aspects of model theory, set theory and combinatorics. Some of the topics to be discussed are dichotomies for orbit equivalence relations (e.g., the Topological Vaught Conjecture, the Glimm-Effros type dichotomies, etc.), non-classification theorems, including the theory of turbulence and its applications, and the relationship of the topological dynamics of automorphism groups of countable structures with the Fraïssé construction and Ramsey theory.

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# Plenary Talks

**Grigori Mints,** ***Intuitionistic Frege Systems are Polynomially equivalent***

**Alberto Marcone,** ***Logical aspects of wqo theory***

**Frank Stephan,** ***Kolmogorov complexity***

**Michael Möllerfeld,** ***On the Proof Theory of $\Pi^1_2$-Comprehension***

**Göran Sundholm,** ***The METAlinguistic Turn***

**Matthew Valeriote,** ***The Constraint Satisfaction Problem and Universal Algebra***

**Tomek Bartoszynski,** ***Measure and Category from set-theoretic viewpoint***

**Guglielmo Tamburrini,** ***Computation and the explanation of intelligent behaviours: ethologically motivated restart***

**Alessandro Berarducci,** ***Definable groups in o-minimal structures***

**John T. Baldwin,** ***Quasiminimal Sets and Categoricity in Abstract Elementary Classes***

**W. Hugh Woodin,** ***Extender models and the inner model problem for supercompact cardinals and beyond***

GRIGORI MINTS

**Intuitionistic Frege Systems are Polynomially equivalent**

joint work with Arist Kozhevnikov

A Frege system for a given propositional logic is a standard formulation with axioms and modus ponens plus maybe other schematic rules. A basic bootstrapping result for investigating NP and co-NP completeness (due to S. Cook and R. Reckow) is polynomial equivalence of every two Frege systems for classical logic. The proof used in an essential way the fact that every classically admissible schematic rule $A/B$ is derivable: $A \to B$ is a tautology. The equivalence of admissibility and derivability fails for the intuitionistic logic that bears the same relation to PSPACE as classical logic to NP.

It is possible however to simulate polynomially (in a Frege system) every intuitionistically admissible schematic rule stated with standard connectives $\&, \vee, \neg, \to$. Therefore any two intuitionistic Frege systems polynomially simulate each other.

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Alberto Marcone

**Logical aspects of wqo theory**

A quasi-ordering (i.e. a reflexive and transitive binary relation) is a wqo (well quasi-ordering) if it contains no infinite descending chains and no infinite sets of pairwise incomparable elements.
The existence of several equivalent definitions of wqo witnesses that the notion is very natural. We study the strength of the equivalences among five different definitions. From the viewpoint of reverse mathematics we are dealing with fairly weak subsystems of second order arithmetic, since each equivalence is provable assuming either Ramsey Theorem for pairs or Weak König Lemma. By constructing computable counterexamples we establish which equivalences hold effectively and which ones are provable in $\mathsf{WKL}_0$. However most of these counterexamples do not lead to reversals in the usual reverse mathematics style.
The closure properties of wqos under many operations have also been investigated from the same viewpoint. Here we study statements such as "the product of two wqos is a wqo".
A strengthening of wqo is the notion of bqo (better quasi-ordering), which was introduced by Nash-Williams in the 1960's and has proved to be very useful for showing that a quasi-ordering is indeed a wqo. Even the simplest facts of bqo theory require fairly strong axioms, and not much is provable below $\mathsf{ATR}_0$.

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Frank Stephan

## Kolmogorov complexity

Kolmogorov complexity measures the size of an object by considering the length of the shortest program that generates this object. Three approaches to determine the complexity of nonrecursive sets and functions are considered:

- Measures for the local complexity. What is, for a set $A$, the worst case conditional Kolmogorov complexity of $(A(x_1), \ldots, A(x_n))$ given $x_1, \ldots, x_n$?
- Randomness notions for sets and their characterization in terms of the Kolmogorov complexity of initial segments $A(0)A(1)\ldots A(n)$.
- Turing degrees and reducibility.

The historic background of these notions is given and the following major topics and results are addressed.

- If the local complexity of the Kolmogorov function $x \to C(x)$ is constant relative to $A$ then $A \geqslant_T K$.
- Martin-Löf random sets and $\Pi^0_1$ classes.
- The class of random-low sets and its natural characterizations.
- Separating randomness notions in high Turing degrees.
- Martin-Löf randomness relative to the halting problem.
- Kolmogorov complexity and effective Hausdorff dimension.

Concluding remarks deal with still open questions. One major question is whether Martin-Löf randomness can be characterized in terms of nonmonotonic recursive martingales.

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Michael Möllerfeld

## On the Proof Theory of $\Pi^1_2$-Comprehension

We are going to present various formal systems based on notions coming from different areas of logic which turn out to be proof-theoretically equivalent to the formal system of second order arithmetic called $\Pi^1_2$-comprehension.

Among those are weak systems of set theory enriched by axioms claiming the existence of certain stable and strong reflecting sets, weak systems of second order arithmetic enriched by determinacy axioms and generalized monotone quantifiers, and systems of generalized inductive definitions including the $\mu$-calculus, a prominent system in theoretical computer science.

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GÖRAN SUNDHOLM

## The METAlinguistic Turn

A famous phrase of Gustav Bergmann's — *The Linguistic Turn* — has been used to characterise the philosophical shift that was inaugurated a century ago in the writings of Frege, Russell and Wittgenstein. Philosophical questions were now approached in terms of the language used to formulate them and their answers. Also in logic a similar phenomenon takes place, but then thirty years later. The Founding Fathers employed formal languages as a *tool* in foundational studies; a hundred years ago the main task of a logician worth his salt was to design a sizeable mathematical language, adequate for the practice of mathematical analysis after the fashion of Weierstrass, and to provide careful meaning-explanations that make evident the axioms and rules for the system in question. Frege, Whitehead- Russell, Wittgenstein and many adhered to this modern version of the Aristotelian conception of demonstrative science. With the advent of metamathematics in the seminal works of Gödel and Tarski, and its ensuing masterly codification by Bernays the formal language is demoted from being a tool into a mere object of study. In particular, the Fregean assertion-sign, i.e. the turnstile, changes into a theorem-predicate.
The lecture outlines these developments and places this dichotomy, namely Langue *with Content (for use) versus ("object"-) Language for Study* (only) alongside Van Heijenoort's *Logic as Language versus Logic as Calculus* and *Hintikka's Language as the Universal Medium versus Language as Calculus.*

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Matthew Valeriote

## The Constraint Satisfaction Problem and Universal Algebra

An instance of the Constraint Satisfaction Problem (CSP) consists of a finite domain $A$, a finite set of variables $V$ and a finite set of constraints of the form $\langle \vec{s}, R \rangle$ where $\vec{s}$ is a finite sequence of elements from $V$ and $R$ is an $|\vec{s}|$-ary relation over $A$. A solution of the instance is a function $f$ from $V$ to $A$ such that for each constraint $\langle \vec{s}, R \rangle$, $f(\vec{s}) \in R$. It turns out that many combinatorial problems can be expressed in a natural way within the framework provided by CSPs.
In general, CSPs form an NP-complete class, but if one restricts the type of relations that can appear in the constraints, it is possible to obtain tractable classes of the CSP. A central problem is to determine those sets of relations for which the class of corresponding CSPs is tractable. In my talk I will present an approach to solving this problem via universal algebra that has been developed over the past several years.

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Tomek Bartoszynski

## Measure and Category from set-theoretic viewpoint

The notions of Lebesgue measure zero set and of the first category set have been around for over 100 years and they appear in one form or another in nearly every area of mathematics. In this talk I will survey the set-theoretic results concerning these two concepts.
There is a fair number of results concerning measure and category that remain true when words “measure zero” and “first category” are interchanged. These are examples of duality between these two notions. The main focus of my talk will be on properties that break this symmetry, in other words, when the duality fails. These non-duality results often expose combinatorial properties of measure zero and first category sets that are of independent interest and have relevance for analysis and topology.

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Guglielmo Tamburrini

## Computation and the explanation of intelligent behaviours: ethologically motivated restart

joint work with Edoardo Datteri

A distinctive goal of empirical science is to provide understanding and explanation of the natural world. The problem of scientific explanation was often neglected in philosophical reflections on computation and intelligent behaviour that are grounded in logic and the theory of computability. Notably, Turing focussed on imitation, rather than explanation. And Gödel's reflections on incompleteness and mind invoke idealizations that are doubtful in the context of empirical modelling and explanation.
What is the role, if any, of computation in explaining intelligent behaviours? This broad question is explored in connection with adaptive sensorimotor behaviours that involve no intentionality or consciousness. How-does-it-work explanations of these ubiquitous biological behaviours are currently pursued in cognitive ethology, biorobotics, and the cognitive neurosciences. Illustration of this explanatory strategy begins with simple mechanism schemata accounting for phototropic behaviours - from Loeb's moth [2] to Braitenberg vehicles [1]. These schemata posit fairly direct sensorimotor connections, and make no appeal to interposed computation processes. Analogical "stretch-reflex" mechanisms, however, are inadequate to model many (even just insect-level) animal behaviours. Elementary action-selection and planning capabilities are hypothesized in ethological models proposed by Jennings, Tinbergen, Baerends. And computational models, developed within biorobotic frameworks, often enrich reflexive mechanisms with various kinds of discrete decision processes. Mechanism schemata involving case-based reasoning and symbolic planning for action selection are extensively discussed, emphasizing distinctive roles of symbolic computation in the understanding of adaptive behaviours.

### References

[1] V. Braitenberg, ***Vehicles: Experiments in synthetic psychology***, MIT Press, Cambridge, MA, 1984.
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Alessandro Berarducci

## Definable groups in o-minimal structures

We are interested in definable groups in an o-minimal structure $M$, namely groups whose underlying set is definable and with a definable group operation. Such groups have been studied by many authors and the results so far obtained suggest a close analogy between definable groups and Lie groups. In particular it follows from the results of A. Pillay (1988) that if the underlying set of the o-minimal structure is the real line, then every definable group is indeed a Lie group, while Y. Peterzil, A. Pillay e S. Starchenko (2000) obtained matrix representation theorems for definable groups which confirm the analogy with Lie groups. Recently Pillay (2003) stated a conjecture which, if solved positively, would greatly clarify the relationship between Lie groups and definable groups. As a partial answer to the conjecture we prove, in collaboration with M. Otero, Y. Peterzil and A. Pillay, that it is possible to associate in a canonical (and non-trivial) way to every definable groups $G$ a compact Lie group $G/G^{00}$. Here $G^{00}$ is the smallest "type definable subgroup of bounded index" (whose existence is part of the result) and $G/G^{00}$ has a suitable "logic topology" studied by Lascar and Pillay. One would expect that if $G$ is definably compact then $G$ should resemble $G/G^{00}$ very closely (say it is elementary equivalent).

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John T. Baldwin

## Quasiminimal Sets and Categoricity in Abstract Elementary Classes

We distinguish two themes in generalizing Morley's theorem to infinitary logic. One is focused primarily on the logic $L_{\omega_1,\omega}$ and does not assume that there are arbitrarily large models. The other concerns the more general setting of Abstract Elementary Classes (AEC): a class with a notion of strong submodel that satisfies a variant of the Jónsson axioms. In that direction most progress has been made by assuming that the class has arbitrarily large models and the amalgamation property. In this talk, we will focus on the 'upwards' direction in $L_{\omega_1,\omega}$ and $L_{\omega_1,\omega}(Q)$. On the one hand we will place Zilber's study of quasiminimal sets in the general context of Shelah's work on categoricity in $L_{\omega_1,\omega}(Q)$. On the other we consider the possibilities of using this analysis in the study of complex exponentiation. Here, we will review the connection with Hrushovski's extension of the Fraïsse construction to build stable structures.

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W. Hugh Woodin

## Extender models and the inner model problem for supercompact cardinals and beyond

The inner model program within Set Theory concerns the definition and study of canonical models for large cardinal axioms. The current family of such inner models are Extender Models which are constructed from extender predicates. To reach large cardinals beyond the level of Superstrong Cardinals it is necessary to use extender predicates derived from long extenders. However the use of long extenders creates new difficulties in the analysis of the models even granting iterability. The most serious of these difficulties is the moving spaces problem identified by Steel. We discuss the problem and introduce a family of extender models which avoid the problem. Assuming a generalized iteration hypothesis these extender models solve the inner model problem for essentially all known large cardinal axioms.

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# Special Session — Proof Theory

ANTON SETZER

**Universes in Type Theory: Mahlo and $\Pi_3$-Reflection**

This research is part of a programme with the goal of extending Martin-Löf Type Theory (MLTT) in order to reach as much proof theoretic strength as possible, while still keeping constructively justified theories. From a foundational point of view this provides a counterpart of ordinal theoretic proof theory: there the consistency of theories is reduced to the well-foundedness of certain ordinal notation systems. The aim of our programme is to provide constructive theories in which the well-foundedness of those notation systems can be shown. Putting these two steps together one obtains a complete reduction of the consistency of fragments of set theory and analysis, sufficient to prove most mathematical theorems, to constructively justified theories.
From a more applied point of view the goal is to discover new data structures which can be used in general programming. Some of the ideas developed as part of this project (the Mahlo universe) have already been used in order to develop a closed formalisation of inductive-recursive definitions. The use of this in generic programming is currently been explored by Peter Dybjer, Marcin Benke and Patrik Jansson.
In this talk we will first review the main principles of MLTT (universes and the W-type) which already reach the strength of one recursively inaccessible. Then we will look at the Mahlo universe, which reaches the strength of one recursively Mahlo ordinal. Finally we look at how to design a theory the author conjectures to reach the strength of a $\Pi_3$-reflecting-ordinal. If time permits we consider extensions, conjectured to reach the strength of $\Pi_n$-reflection, of $\Pi_\alpha$-reflection and (a weak conjecture) of $\Delta^1_1$-reflection.

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Andrea Cantini

**Classifications, operations and (constructive) sets**

We deal with systems of set theory ranging from weak (non-extensional) consistent versions of naive set theory to constructive set theories, which are proof-theoretically not stronger than Peano Arithmetic. In particular, we consider a system which is proof-theoretically weak but computationally complete (being able to simulate combinatory logic), and a version $\mathsf{CZFU(op)}$ of Aczel's system of constructive set theory, where urelements form an applicative structure. The motivation for the second system is that, constructively, it is natural to have primitive notions of 'number' and '(intensional) operation', which are independent on sets. Technically, we can develop in $\mathsf{CZFU(op)}$ a general theory of inductive definitions (à la Aczel); yet, the consistency strength is kept at the arithmetical level.

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ALAN WOODS

**Probability limit laws for ordinal numbers**

joint work with Andreas Weiermann

The usual scenario with logical limit laws is to consider a class of finite structures — for definiteness, let's say rooted trees. One takes an arbitrary sentence in some logic and tries to show that as $n$ goes to infinity, almost all, almost none, or at least an asymptotically constant fraction of the trees of size $n$ from that particular class satisfy the sentence.
If we consider some class of ordinals $\alpha$, how might the logically definable properties of the typically *infinite* structures $(\alpha, <)$ satisfy a limit law? What is needed is a notion of "finite size" for $\alpha$. When $\alpha < \varepsilon_0$, the full Cantor normal form (CNF) for $\alpha$ provides such a notion. Indeed it is folklore that there is a natural one to one correspondence between finite rooted trees and CNF *notations* for ordinals less than $\varepsilon_0$.
Let $\delta_n^\beta(\varphi)$ be the fraction of ordinals $\alpha < \beta$ of size $n$ for which $(\alpha, <) \models \varphi$.

**Theorem 1.** *For each infinite $\beta \leqslant \varepsilon_0$, $\delta_n^\beta(\varphi)$ converges for all first order sentences $\varphi$. The limit is always either $0$ or $1$, except in the cases:*

*(i)* $\beta < \omega^\omega$ *and* $m_1 > 1$ *in the CNF*

$$\beta = \omega^{k_1} m_1 + \omega^{k_2} m_2 + \cdots + \omega^{k_s} m_s, \quad k_1 > k_2 > \cdots > k_s.$$

*(ii)* $\beta = \varepsilon_0$.

Monadic second order logic is more involved — $\delta_n^\beta(\varphi)$ need not converge for some choices of $\beta$ — but can also be analysed. In at least one case, there is even a limit law for structures $(\alpha, <, +)$.

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ROBERT S. LUBARSKY

**On the Independence of Variants of Power Set under IZF - Power Set**

As is well known, classically equivalent principles can become inequivalent intuitionistically. This phenomenon is demonstrated for variants of the Power Set Axiom.
The variants in question are Power Set itself, Exponentiation (for all sets A, B the collection of functions from A to B forms a set), and Subset Collection (equivalently, Fullness), an axiom discovered by Aczel (see [1], [2], [3]) while developing CZF, a version of Myhill's CST (see [4]). There are soft proofs that Power Set implies Subset Collection, which in turn implies Exponentiation. Rathjen (see [5], [6]) showed via an ordinal analysis that Subset Collection does not imply Power Set under CZF; a slight extension of those ideas shows the same under IZF - Power Set. However, that method could not differentiate Exponentiation and Subset Collection, since the theories in question have the same proof-theoretic ordinal. In this talk, first a new proof that Subset Collection does not imply Power Set is presented, using Kripke models. Then this construction is extended to show that Exponentiation does not imply Subset Collection.

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# Special Session — Model Theory

Ivan Tomašić

## Probabilistic Independence Theorem in pseudofinite fields

Independence Theorem for pseudofinite (and slightly more general) fields was first isolated by Hrushovski. Kim and Pillay's generalisation of this theorem gave rise to the study of simple theories. It says that, given a complete type $p$ over a (small) submodel $K$ and two generic extensions $p_1, p_2$ of $p$ to sets of parameters $F_1$ and $F_2$ independent over $K$, the type $p_1 \cup p_2$ remains generic.
We show how these types can in fact be viewed as independent events in a suitable probability space. This also has a motivic interpretation.

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XAVIER VIDAUX

**The analogue of Büchi's problem in polynomial rings**

joint work with Thanases Pheidas

We solve the analogue of a Diophantine problem of Büchi over rings of polynomials. As a consequence we show the following result in Logic: Let $F$ be a field of characteristic either 0 or $\geqslant 13$. Let $L$ be the first order language which contains a symbol for addition, a symbol for the property '$x$ is a square' and symbols for the elements of the image of $\mathbf{Z}$ in $F$. Then, multiplication is existentially definable over the ring of polynomials $F[t]$, in the language $L_t$, where $L_t$ extends $L$ by a symbol for the variable $t$. Then, from known results, it follows that the Diophantine theory of $F[t]$ is undecidable, in the language $L_t$.

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ALEXANDER BERENSTEIN

**Probability spaces with a generic automorphism**

joint work with C. Ward Henson

We prove that the class of generic structures among those consisting of the measure algebra of a probability space equipped with an automorphism is axiomatizable by positive sentences interpreted using an approximate semantics. The separable generic structures of this kind are exactly the ones isomorphic to the measure algebra of a standard Lebesgue space equipped with an aperiodic measure-preserving automorphism. We show that the corresponding theory is complete and has quantifier elimination; moreover it is stable with built-in canonical bases. We give an intrinsic characterization of its independence relation.

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Tamara Servi

**On the decidability of the real exponential field: candidates for a complete axiomatization**

In the paper [1], Macintyre and Wilkie proved the decidability of the structure $\mathbb{R}_{\text{exp}} = \langle \mathbb{R}; +, -, \cdot, <, \exp, 0, 1 \rangle$, under the assumption that the Real Schanuel Conjecture (RSC) is true. RSC says that given $a_1, ..., a_n \in \mathbb{R}$, the transcendence degree (over $\mathbb{Q}$) of $\langle a_1, ..., a_n, e^{a_1}, ..., e^{a_n} \rangle$ is greater or equal to the linear dimension (over $\mathbb{Q}$) of $\langle a_1, ..., a_n \rangle$. In the paper [2], Berarducci and Servi proved that $\mathbb{R}_{\text{exp}}$ is effectively o-minimal: there are recursive bounds on the number of connected components of $\mathbb{R}_{\text{exp}}$-definable sets. This result allows the authors to simplify the axiomatization proposed in [1] and to present new candidates for a complete axiomatization of $\mathbb{R}_{\text{exp}}$. Moreover, the results can be generalized to obtain similar decidability arguments (conditional on suitable conjectures) for some related structures (i.e. pfaffian expansions of the real field).

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## Raf Cluckers

### Ax-Kochen-Eršov principles for integrals over local fields

We present some joint work with F. Loeser, L. Lipshitz, and Z. Robinson on extensions of classical results à la Ax-Kochen-Eršov to $p$-adic and $\mathbb{F}_p((t))$- integrals in a motivic framework. Instead of comparing the truth value of sentences in $\mathbb{Q}_p$ and $\mathbb{F}_p((t))$ for $p$ sufficiently big, we compare the volume of definable subsets of $\mathbb{Q}_p^n$ and $\mathbb{F}_p((t))^n$, not only in the language of valued fields, but also in a language containing an angular component map and an analytic structure. The content of this talk will be covered in [1] and [2].

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Monica VanDieren

## Shelah's Categoricity Conjecture for Tame Abstract Elementary Classes

We prove a generalization of Morley's Theorem for a broad range of non-elementary classes. This captures some of the most interesting and natural cases of Shelah's 30 year old categoricity conjecture for $L_{\omega_1,\omega}$ which include excellent classes. The new component of our work is an upward categoricity transfer theorem. Earlier attempts to prove upward categoricity transfer results for non-elementary classes relied either on syntax (and compactness) or extra set-theoretic assumptions. Our results for tame abstract elementary classes hold in ZFC. This is joint work with Rami Grossberg.

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Karim Zahidi

**Elimination theory for addition and Frobenius in certain rings of positive characteristic**

We will discuss some recent results concerning the theory of addition and the Frobenius map in polynomial rings of positive characteristic. In particular we will prove a relative (relative with respect to the theory of the ground field) model completeness result for this theory. If time permits, we will also report on work in progress, concerning the theory of addition and Frobenius in rings of formal power series.

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MATTHIAS ASCHENBRENNER

**Faithfully flat Lefschetz extensions**

joint work with Hans Schoutens

We call a commutative ring of characteristic zero a Lefschetz ring if it is (isomorphic to) an ultraproduct of rings of prime characteristics. It is well-known that the field $\mathbb{C}$ of complex numbers is Lefschetz. In this talk I will discuss the following question: which Noetherian rings of characteristic zero admit a faithfully flat embedding into a Lefschetz ring? It is well-known that any polynomial ring over $\mathbb{C}$ admits such an embedding (van den Dries and Schmidt). I will show that each Noetherian local ring of equicharacteristic zero admits a faithfully flat Lefschetz extension, which is even functorial in a way. I will then briefly indicate some applications to commutative algebra.

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# Special Session — Universal Algebra

Paolo Aglianò

## Basic-algebras: an overview

Basic algebras have been introduced by P. Hájek in order to investigate many-valued logic by algebraic means. Hájek's motivations for introducing basic algebras were of two kinds. The first one was providing an algebraic counterpart of a logic, called Basic Logic, which embodies a fragment common to the most important many-valued logics, namely Łukasiewicz Logic, Gödel Logic and Product Logic. The second one was to provide for an algebraic mean for the study of continuous $t$-norms on $[0, 1]$.

Basic algebras also have a purely algebraic root, namely the theory of residuated monoids and of hoops in particular. A **hoop** is a commutative, residuated, integral, partially ordered monoid $\langle A, \cdot, \rightarrow, 1\rangle$ in which the equation $x(x \rightarrow y) \approx y(y \rightarrow x)$ holds. Hoops have been introduced by J.R. Büchi and T.M. Owens and typical examples of hoops arise from classical algebra. For instance the positive cone of a lattice-ordered abelian groups, and the set of two-sided ideals of a ring can be given a natural hoop structure. However hoops have also a logical interest, in that they constitute the algebraic counterpart of fragments without negation and $\perp$ of non-classical logics.

It turns out that basic algebras are precisely the bounded **basic hoops**, i.e. the bounded hoops that are isomorphic with subdirect products of totally ordered hoops. Thus the general algebraic theory of hoops applies to basic algebras as well. In particular, the theorem on the decomposition of subdirectly irreducible hoops as an ordinal sum of a hoop and a nontrivial subdirectly irreducible Wajsberg hoop naturally extends to basic algebras. This result suggests that the fundamental structures in the study of basic algebras are Wajsberg hoops and the fundamental operation is the ordinal sum. This intuition led to an interesting, namely: *every totally ordered (hence a fortiori every subdirectly irreducible) basic algebra is an ordinal sum of a family of Wajsberg hoops.*

Using this result as a tool we shall give an overview of the ongoing research on the subject, focusing mainly on subvarieties of the variety of basic algebras.

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Vera Vértesi

## Checking equations in finite algebras

Identities play an important role both in universal algebra and complexity theory. In this talk identities will be studied from different viewpoints concerning to these fields.

The *identity checking problem* asks whether two given terms over an algebra agree for every substitution. J. Lawrence and R. Willard [1] had previously shown the coNP-completeness of the identity checking problem for matrix rings large enough to have a nonsolvable group of units. Last year with Cs. Szabó [2] we proved—by using only "ordinary" semigroup words—that the identity checking problem is coNP-complete for all other matrix semigroups: $M_2(\mathbb{Z}_2)$ and $M_2(\mathbb{Z}_3)$. We analyze the same problem for finite rings: Is the identity checking problem coNP-complete for its multiplicative semigroup?

On the other hand the *membership problem* can be checked by equation testing. In some sense the *$\beta$-function* or *equational bound* is the measure of the complexity of the membership problem. $\beta(n)$ is the minimum length of such equations that are sufficient to check to determine whether an $n$-element algebra is in the variety or not. This function exists and it is uniquely determined for any variety, and it is recursive (it can be algorithmically computed). By the construction of the free-algebra one can show that the $\beta$-function is at most double-exponential. Z. Székely [3] found an algebra for which the $\beta$-function is sublinear. In this talk we investigate the $\beta$-function for hypergraph algebras, and prove that in general it is not bounded by any polynomial.

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Steve Givant

**Interconnections between groups and algebras of relations**

The calculus of binary relations — an important forerunner of first-order logic — was created by De Morgan, Peirce, and Schröder in the second half of the nineteenth century. In the period 1941–43, Tarski recast the calculus as an abstract, finitely axiomatized, equational theory of *relation algebras.* One of his original goals was to show that all abstract relation algebras are representable as (that is, isomorphic to) concrete algebras of (binary) relations. Lyndon showed that this goal is unattainable, but Jónsson and Tarski, and later Maddux, did establish important representation theorems for restricted classes of relation algebras. In this talk we present the following substantial generalization of the theorems of Jónsson-Tarski and Maddux.

A relation algebra is said to be *measurable* if its identity element is the sum of measurable atoms — atoms $x$ whose "size" can be determined. (Roughly speaking, this is done by counting the number of "functional" elements below the "square" with side $x$.) A concrete construction of measurable relation algebras is introduced, one that generalizes the well-known construction of the complex algebra of a group. Instead of a single group, the construction involves a system of pairwise disjoint groups, a corresponding system of isomorphisms between quotients of the groups, and a system of cosets in the quotients that serve to "shift" relational composition. The resulting class of *generalized group relation algebras* includes all (full) algebras of relations and also many interesting new examples of non-representable relation algebras. The following theorem shows that the concrete construction yields all possible examples of measurable relation algebras.

**Theorem.** *Every atomic, measurable relation algebra is essentially isomorphic to a generalized group relation algebra.*

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Benoit Larose

## Some results on the complexity of strong colourings of hypergraphs with fixed group of symmetry

Let $G$ be a subgroup of the symmetric group $S_h$. The *strong colouring problem for* $G$ ($SCP_G$), asks, for a given $h$-ary irreflexive relation $\rho$ with group of symmetry $G$, if there is a strong colouring of $\rho$. We investigate the complexity of this constraint satisfaction problem using universal algebraic techniques. In particular, we answer a question posed by Haddad and Rödl in 1992 by exhibiting (an infinite family of) examples of a group $G$ with subgroup $H$ such that $SCP_G$ in in $\mathbf{P}$ but $SCP_H$ is $\mathbf{NP}$-complete.

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Paolo Lipparini

## Tolerance intersection properties

R. Freese and B. Jónsson showed that if every subalgebra of $\mathbf{A}^2$ is congruence modular then $\mathbf{A}$ is arguesian. Can we improve "arguesian" to "higher arguesian" in M. Haiman's sense? For congruence varieties the answer is yes [1].

**Theorem.** *(a) If every subalgebra of* $\mathbf{A}^2$ *is congruence modular then:*
(wTIP) $\alpha \cap \Theta^* = (\alpha \cap \Theta)^*$, *for every congruence* $\alpha$ *and tolerance* $\Theta$ *of* $\mathbf{A}$, *where* $^*$ *denotes transitive closure.*
*(b) If every subalgebra of* $\mathbf{A}^4$ *is congruence modular then:*
(wTIP$^2$) $\Gamma^* \cap \Theta^* = (\Gamma \cap (\Theta \circ \Theta))^*$, *for all tolerances* $\Theta$ *and* $\Gamma$ *of* $\mathbf{A}$.

wTIP is equivalent to H. P. Gumm's Shifting Principle.
By (a) we can apply the methods of [1] to the class of *simple terms*, that is the smallest class (of terms in the language of lattices) which (i) contains all variables, and (ii) contains all terms of the form $x_i \wedge (t_1 \vee t_2 \vee \cdots \vee t_n)$, whenever $n > 0$, $x_i$ is a variable, and $t_1, t_2, \ldots t_n$ are simple terms.
Detailed proofs can be found at the author's web page.

**Problems.** (1) Do wTIP or wTIP$^2$ imply the arguesian identity (or even the higher arguesian identities)?
(2) Is there any $n$ such that if every subalgebra of $\mathbf{A}^n$ is congruence modular then $\mathbf{A}$ satisfies all (some of) the higher arguesian identities (or even TIP [1])?
(3) Can the arguesian identity (the higher arguesian identities) be written as $p \leqslant q$, where $p$ is simple?

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Erhard Aichinger

## The variety of near-rings is generated by its finite members

Certain varieties of algebras, such as the variety of groups or the variety of semigroups, are generated by their finite members, others, such as the variety of modular lattices [Fre79], are not. In this talk, we study the variety of near-rings.

A near-ring [Mel85, Pil83] is an algebra $\mathbf{N}$ with operations $+$ (binary), $-$ (unary), $\mathsf{0}$ (nullary), and $\circ$ (binary) such that $\langle N; +, -, \mathsf{0}\rangle$ is a (not necessarily abelian) group, and $\mathbf{N}$ satisfies the identities $\forall x \forall y \forall z\, ((x+y)\circ z \approx x\circ z + y \circ z)$ and $\forall x \forall y \forall z\, ((x\circ y)\circ z \approx x \circ (y \circ z))$. We will show that the variety of near-rings is generated by its finite members. Let $\mathcal{N}$ be the variety of all near-rings, and let $\mathcal{N}_{\text{fin}}$ be the class of finite near-rings. Then using the class operators $\mathbb{H}$, $\mathbb{S}$, $\mathbb{P}$ (see [MMT87, Definition 4.91]), our result can be written as

$$\mathbb{HSP}(\mathcal{N}_{\text{fin}}) = \mathcal{N}.$$

The idea of the proof is to show that every identity that fails in some near-ring even fails in some finite near-ring.

We will show that the techniques can be applied not only to near-rings, but, more generally, to varieties of "composition algebras".

The results presented in this talk are going to appear in [Aic00].

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Lászlo Zádori

**Bounded width problems and algebras**

joint work with Benoit Larose

We investigate the class of bounded width CSPs (constraint satisfaction problems) for relational structures. This special class of polynomial-time CSPs was introduced by Feder and Vardi in a 1999 paper. In our study of bounded width CSPs we assign an algebra to every relational structure. We show that if the CSP for the structure has bounded width then the variety generated by the algebra omits the Hobby-McKenzie types 1 and 2. We conjecture that the converse of this statement also holds.
In their paper Feder and Vardi proved that the CSP for any structure is polynomial-time equivalent to the retraction problem for a poset obtained by a certain procedure from the structure. We call this poset the Feder-Vardi poset of the structure. We show that if the retraction problem for the Feder-Vardi poset has bounded width then the CSP for the structure also has bounded width. We do not know if the converse of this claim is true.

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MIKLÓS MARÓTI

**Finite basis problems and results for quasivarieties**

joint work with Ralph McKenzie

We offer new proofs and generalizations of two important finite basis theorems of D. Pigozzi and R. Willard. Our proofs rely on the study of several congruence properties for quasivarieties, including having pseudo-complemented congruence lattices (PCC) and the weak extension principle (WEP). We show that congruence meet semi-distributive varieties have PCC, and relatively congruence distributive quasivarieties have both the PCC and the WEP.

**Definition.** For any quasivariety $\mathcal{K}$, we write $\mathcal{K} \models \mathrm{PCC}$ to denote that for all $\mathbf{A} \in \mathcal{K}$, $\mathbf{Con\,A}$ is a pseudo-complemented lattice, i.e., for every congruence $\alpha$ of $\mathbf{A}$, there is a largest congruence $\delta$ with the property $\alpha \wedge \delta = 0_A$.

**Theorem.** *Every quasivariety of finite signature, contained in a finitely generated quasivariety and having pseudo-complemented congruence lattices, is contained in a finitely axiomatizable, locally finite, quasivariety.*

**Definition.** For quasivariety $\mathcal{K}$, we write $\mathcal{K} \models \mathrm{WEP}$ to denote that $\mathcal{K}$ has the week extension principle, i.e., for all $\mathbf{A} \in \mathcal{K}$ whenever $\phi, \theta \in \mathrm{Con}\,\mathbf{A}$ and $\phi \wedge \theta = 0_A$ then $\phi' \wedge \theta' = 0_A$, where the map $' :\ \mathrm{Con}\,\mathbf{A} \to \mathrm{Con}_{\mathcal{K}}\mathbf{A}$ is the extension map defined by $\alpha' = \bigcap\{\gamma \in \mathrm{Con}_{\mathcal{K}}\mathbf{A} : \alpha \leqslant \gamma\}$.

**Theorem.** *Let $\mathcal{K}$ be a finite set of finite algebras of the same finite signature such that $SP(\mathcal{K}) \models \mathrm{PCC}$. If $HS(\mathcal{K}) \subseteq SP(\mathcal{K})$ then $SP(\mathcal{K})$ is finitely axiomatizable.*

**Theorem.** *Every finitely generated quasivariety of finite signature which satisfies the WEP and the PCC is finitely axiomatizable.*

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# Contributed Papers

**Anatoly P. Beltiukov**, *Polynomial Time Programming Language*

**Stefano Berardi**, *Classical Arithmetic as solution of a Maximum problem*

**Josef Berger**, *Unique Existence Theorems in Constructive Analysis*

**Marta Bilkova**, *A computational view of intuitionistic propositional proofs*

**Stefan Bold**, AD *and Descriptions for cardinals below* $\Theta$

**Roger Bosch**, *Definably large cardinals*

**Andrey I. Bovykin**, *Several proofs of* $PA$*-unprovability*

**Jens Brage**, *A natural interpretation of classical proofs*

**Riccardo Bruni**, *A Note on Gödel's Philosophy of Mathematics in the Light of His* Nachlass

**Dumitru Buşneag**, *Localization for some algebras of fuzzy logic*

**Anahit A. Chubaryan**, *On the proofs complexity in the Resolution systems of Intuitionistic and Minimal propositional logic*

**P. Cintula**, *Towards Universal Fuzzy Logic*

**Camillo Costantini**, *Consistent examples about extensions of continuous functions*

**Laura Crosilla**, *On constructing completions*

**Paola D'Aquino**, *Quadratic reciprocity law in weak fragments of* $PA$

**L. Darniere**, *Model-completions of scaled lattices*

**Jairo José da Silva**, *On the principle of excluded middle*

**Dejan Delić**, *Finite Decidability of Locally Finite Varieties*

**Alexander Denisov**, *Successful Modification of the Nuclear Theory*

**Mauro Di Nasso**, *Different paths to nonstandard analysis*

**Natasha Dobrinen**, *Almost everywhere domination*

**Christoph Duchhardt**, *Reflection vs Thinning*

**Philip Ehrlich,** *The Rise of non-Archimedean Mathematics and the Roots of a Misconception*

**Fredrik Engström,** *Notions of resplendency for logics stronger than first-order logic*

**Horacio Faas,** *Visual and Heterogeneous Inferences*

**Josep Maria Font,** *New forms of the Deduction Theorem and Modus Ponens*

**Andrey Frolov,** *Computable copies of distributive lattices with relative complements and linear orderings*

**Nikolaos Galatos,** *Translations in substructural logic*

**Çiğdem Gencer,** *Unification and Passive Inference Rules for the Modal Logic* $K4$

**Paolo Gentilini,** *Critical Deduction Chains and Inferential Models for Type Theory*

**Amélie Gheerbrant,** *Recursive complexity of L-modal logic*

**Matthew B. Giorgi,** *A High Noncuppable Enumeration Degree*

**Mariusz Grech,** *The automorphisms of the lattice of equational theories of commutative semigroups*

**Nicolas Guzy,** *Topological differential structures*

**Kai Hauser,** *The Philosophical Roots of Cantorian Set Theory*

**Christoph Heinatsch,** *The determinacy strength of* $\Pi^1_2$*-comprehension*

**Csaba Henk,** *A Qualitative Approach to the Church-Turing Thesis*

**James Hirschorn,** *On iterated (proper) forcing without adding reals*

**Albert Hoogewijs,** *Formath: Higher-order logic revised*

**Rostislav Horčík,** *Strong standard completeness of* $\Pi$*MTL*

**Gábor Horváth,** *Identities of finite groups*

**Hykel Hosni,** *Rationality as conformity*

**Bernhard Irrgang,** *Higher-Gap Morasses and Morass Constructions*

**Emil Jerabek,** *Bounded arithmetic in 3-valued logic*

**Vladimir Kanovei**, *Fully saturated extensions of standard universe*

**Asylkhan Khisamiev**, *Quasiresolvable periodic Abelian groups*

**Andrzej Kisielewicz**, *A new solution to old paradoxes*

**Bernhard Koenig**, *Partial Orderings with Dense Subtrees*

**Nurlan Kogabaev**, *Computable lists of $I$-algebras and complexity of some problems*

**Juha Kontinen**, *On definability of second order generalized quantifiers*

**Marcin Kozik**, *A finite algebra with PSPACE-hard membership problem and at least exponential $\beta$ function*

**Jan Kraszewski**, *On some properties of ideals on the Cantor space*

**Mathis Kretz**, *Deduction Chains for Logic of Common Knowledge*

**Krzysztof Krupiński**, *A special thin type*

**Oliver Kullmann**, *The combinatorics of negation patterns*

**Gábor Kun**, *On the fundamental group of finite posets*

**Miloš S. Kurilić**, *Ultrafilters in models of set theory*

**Marcin Kysiak**, *Nonmeasurable algebraic sums of sets of reals*

**Gyesik Lee**, *Kanamori-McAloon principle and the fast growing Ramsey functions*

**Giacomo Lenzi**, *Axiomatizing bisimulation quantifiers*

**Stefano Leonesi**, *On The Boolean Algebras of Definable Sets in Weakly O-minimal Theories*

**Thierry Libert**, *Topological solutions to the Fregean problem and their corresponding alternative set theories*

**Sonia L'Innocente**, *Minimalities and modules over rings related to Dedekind domains*

**Alexandre A. Lyaletsky (Jr.)**, *On types of continuity defined by partially ordered sets and their interconnection with usual topological continuity*

**Maria Emilia Maietti**, *Relative topology in constructive mathematics*

**Krzysztof Majcher**, *Absolutely Ubiquitous Groups*

**Hamid Reza Maleki,** *An Algorithm for Solving Multiobjective Linear Programming*

**Gian Arturo Marco,** *Invariant model theory and automorphism group actions*

**Petar Markovic,** *Star-linear varieties of groupoids*

**Ralph McKenzie,** *Finite decidability and generative complexity in equational classes*

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**H.G.R. Millington,** *Mathematical Foundations in a Lambda Calculus Extension of Łukasiewicz Logic*

**Ryszard Mirek,** *Local Finiteness and Substructural Algebras*

**Erich Monteleone,** *Thoughts in the age of computer science*

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**Marcin Mostowski,** *Arithmetic of coprimality in finite models*

**Nebojša Mudrinski,** *Some Results on Endomorphism Monoids of Countable Homogeneous Graphs*

**Pavel Naumov,** *On Modal Logic of Deductive Closure*

**Sara Negri,** *A uniform cut elimination theorem for contraction-free systems of modal logic*

**Hassan Mishmast Nehi,** *A Canonical Representation for the Solution of Fuzzy Linear System and Fuzzy Linear Programming*

**A.M. Nurakunov,** *On lattices of varieties of algebras*

**Ivonne Pallares Vega,** *Model Theory and Philosophy of Language*

**Giovanni Panti,** *Substitutions as dynamical systems*

**Franco Parlamento,** *Truth in V for* $\exists^*\forall\forall$*-sentences is decidable*

**Jacek Paśniczek,** *A useful reformulation of first-order classical logic*

**Matti Pauna,** *On pressing down game and Banach–Mazur game*

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**Michael Pinsker,** *Set theory in infinite clone theory*

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**S. Yu. Podzorov,** *Rogers semilattices of arithmetical numberings*

**Vadim Puzarenko,** *On computability in structures*

**Cédric Rivière,** *Dimension function in the theory of closed ordered differential fields*

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**Giuseppina Ronzitti,** *On the philosophy of intuitionistic mathematics*

**V.V. Rybakov,** *First-Order Theories of Free Temporal Algebras*

**Pavel Schreiner,** *Automatical recognition of the pretabularity and tabularity in superintuitionistic and positive propositional logics*

**Peter Schuster,** *The projective spectrum as a distributive lattice*

**Marina Semenova,** *Embedding lattices into convex geometries*

**Dmitrij Skvortsov,** *An Extension of Quantified Dummett's Logic that is Complete with respect to Kripke Sheaves*

**Alexey I. Stukachev,** *On structures decidable in admissible sets*

**Dariusz Surowik,** *The different methods of rejection of the thesis of determinism in temporal logic systems*

**Boža Tasić,** $HSP \neq SHPS$ *for Commutative Rings with Identity*

**Laura Tesconi,** *A Strong Normalization Theorem for natural deduction with general elimination rules*

**Nikolaus Thiel,** *Mahlo & Veblen*

**Carlo Toffalori,** *Superdecomposable pure injective modules over integral group rings*

**Tero Tulenheimo,** *Independence-Friendly Modal Logic*

**Luis Adrian Urtubey,** *Vagueness, Logic and Information Processing*

**Alex Usvyatsov,** *Categoricity in complete metric spaces*

Yde Venema, *McNeille completions of lattice expansions*

Jan von Plato, *Consistency of arithmetic through sequent calculus in natural deduction style*

V. Vuksanovic, *A canonical partition theorem for trees*

Agatha C. Walczak-Typke, *Algebraic Structures in Set Theory without the Axiom of Choice*

Andreas Weiermann, *Does (concrete) mathematics matter to Gödel's theorem?*

Roman Wencel, *A genaralization of o-minimality for expansions of Boolean algebras*

Gunnar Wilken, *Characterizing Closure Properties of Ordinals*

Teruyuki Yorioka, *Gap spectra under Martin's Axiom*

Richard Zach, *Gödel's First Incompleteness Theorem and Detlefsen's Hilbertian Instrumentalism*

Ernst Zimmermann, *Natural Deduction with Single Assumptions*

Enrico Zoli, *On Schmidt's $\sigma$-ideal*

Areski Nait Abdallah

## An algebraic approach to commonsense reasoning

We aim at generalizing Curry-Howard isomorphism to reasoning with partial information [2]. Let $\Lambda$ be the set of typed $\lambda$-terms, $\Gamma$ a context and $\varphi$ a formula. Define the *amplitude of* $\varphi$ *in* $\Gamma$ as $\langle \varphi \mid \Gamma \rangle = \{t \in \Lambda \mid \Gamma \vdash t : \varphi\}$. A natural deduction system may then be seen as defining a system of equations on amplitudes. For handling missing premisses in reasoning with partial information, one introduces a set $\Xi$ of metavariables ranging over inhabitants, thus generalizing $\Lambda$ to a set $\Lambda^{\Xi}$ of formal $\lambda$-terms, and contexts to extended contexts $\overline{\Gamma}$. *Formal amplitudes* are similarly defined: $\langle\!\langle \varphi \mid \overline{\Gamma} \rangle\!\rangle = \{t \in \Lambda^{\Xi} \mid \overline{\Gamma} \vdash t : \varphi\}$. Formal amplitudes are then made plausible by ensuring that the metavariables verify certain *acceptance equations,* whose solutions yield then *soft* inhabitants for the tentative conclusions envisioned.

**Example:** "*Birds typically fly unless they are penguins. Tweety is a bird.*" corresponds to extended context $\overline{\Gamma} = \{A : b \to (\neg p)^\star \to f, x : b, \xi : (\neg p)\star\}$, with formal amplitude $\langle\!\langle f \mid \overline{\Gamma} \rangle\!\rangle \ni Ax\xi$, and acceptance equation

$$\xi = \ \mathsf{if}\ \langle\!\langle \mathbf{False} \mid \overline{\Gamma}, y : \neg p \rangle\!\rangle = \mathbf{0}\ \mathsf{then}\ \ \Omega^{(\neg p)^\star}\ \mathsf{else}\ \ \mathbf{0}\ \mathsf{fi}$$

if one adopts e.g. Doyle's TMS [1] point of view. This yields *soft* inhabitation claim $Ax\Omega^{(\neg p)^\star} : f$.

This constructive approach allows "commonsense reasoning" to be seen as a methodology for solving certain algebraic equations on formal amplitudes.

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Mojtaba Aghaei

**A lambda Calculus for basic propositional logic**

Basic propositional logic was first introduced by A. Visser in 1981, by the intention of formalizing provability. Formal provability logic is related to the provability modal logic $GL$ just as intuitionistic logic is related to modal logic $S4$. Basic logic itself is related to $K4$. It is sound and complete with respect to the class of transitive Kripke models.
Basic Predicate Calculus, $BQC$, was introduced by W. Ruitenburg in 1990 and his motivation came from a philosophical criticism of the BHK (Brouwer-Heyting-Kolmogorov) interpretation of the logical connectives. In this lecture we present a lambda calculus for formalizing motivations of basic logic.

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SEMIHA AKINCI

## David Lewis's Counterpart Theory

Lewis develops counterpart theory in detail in his paper "Counterpart Theory and Quantified Modal Logic". In this paper he argues that we can formalize our modal discourse without using modal operators, that is, a modal system can be developed within ordinary quantification logic with identity. He calls this system Counterpart Theory. In this paper, I compared Lewis's Counterpart Theory with Actualism and Transworld Identity and this theory of counterparts must be revised in important respects. I can think of two directions for attempted revision: either to defend a form of essentialism or to base the notion of counterparts on intentional contexts.

Key Words: Counterpart Theory, Actualism, Possible Worlds Semantics, Modal Logic, Ontology, Symmetry, Reflexivity, Transitivity.

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Pavel E. Alaev

**Strongly computable Boolean algebras**

A computable structure $\mathfrak{M}$ is said to be strongly computable if its complete diagram (of first order) is computable, and to be n-computable if its $\Sigma_n$-diagram is computable. $\mathfrak{M}$ is computable iff it is 0-computable. In other words, $\mathfrak{M}$ is strongly computable if, given a first-order formula and a tuple of elements, we can effectively determine whether this tuple satisfies the formula.

If T is a first-order theory then let $\mathrm{cm}(T) = \inf\{n \in \omega \mid$ every $n$-computable structure $\mathfrak{M} \models T$ has a strongly computable isomorphic copy$\}$. In this talk, we completely describe $\mathrm{cm}(T)$ for every $T$ which is the theory of some class of Boolean algebras, i.e., for every $T$ that extends the axioms of Boolean algebras.

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VAHID AMIRZADEH

**Process Capability in Fuzzy Quality Control**

joint work with Mashaalah Mashinchi and Hamid Reza Maleki

In most quality control systems, there are different indices for evaluation of process capability; natural tolerance limits are compared with specification limits by some criterion.occasionally it is necessary that process capability is evaluated using a customer‘s opinion; in this case we use fuzzy quality control and membership function involving these opinions. In this paper we introduce a new index which computes the process capability based on a membership function.

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Yuuki Andou

## A representation of essential reductions in sequent calculus

We investigate some features of the cut-elimination procedure in sequent-calculus [2] in the viewpoint of the normalization procedure in natural deduction [3][4], using the interpretation of LK to NK defined in our previous work [1]. In order to find the essential reductions of natural deduction in sequent calculus, we classify the structural steps of cut-elimination procedure.

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Alireza Arabpour

**Fuzzy Linear Regression Model with Numerical Input and Fuzzy Output**

joint work with Hamid Reza Maleki and Mashaalah Mashinchi

Fuzzy linear regression models are used to obtain a best linear relation, between a dependent variable and several independent variables, in a fuzzy environment. Several methods for evaluating fuzzy coefficient in linear regression models, are proposed. In this paper, we discuss a fuzzy linear regression model with numerical input and fuzzy output data. We use the concept of comparison fuzzy numbers, to obtain a best model by linear programming. The methodology is illustrated by a numerical example.

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Toshiyasu Arai

**Proofs of the termination of rewrite rules for polytime functions**

joint work with Georg Moser

In [2] a clever term rewriting characterization of the polytime functions is given by defining a feasible rewrite rules for predicative recursion in [3]. It is shown that the derivation length function for a polytime function is bounded by a monotone polynomial in the length of the inputs. We give a simplified proof of this result. As a consequence we obtain the stronger result that the derivation length function depends only on the length of the *normal* inputs. Specifically we give an interpretation $\mathrm{S}(t) < \omega$ of ground terms $t$ which is compatible with the feasible rewrite rules.

Moreover an inspection on the interpretation $\mathrm{S}(t)$ yields an ordinal interpretation $\pi(t)$ of ground terms $t$, where $\pi(t)$ is an ordinal term over the alphabet $0, +, \varphi_n\,(n < \omega)$ with the fixed point free Veblen functions $\varphi_n$. This enjoys the condition that $\pi(s) <_k \pi(t)$ if $s$ is a contractum of $t$ under the feasible rewrite rules, and the number $k$ depends only on the term $t$. The relation $\alpha <_k \beta$ is a subrelation of $\alpha < \beta$ such that the maximal length of the descending $<_k$-chain begining with $\varphi_m n$ is bounded by a polynomial of $n = \varphi_0 0 + \cdots + \varphi_0 0$ for each $k$ and $m$.

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Mohammad Ardeshir

## Basic Arithmetic

Basic Arithmetic, **BA** is the basic logic, **BQC** equivalent of Heyting Arithmetic over intuitionistic logic and of Peano Arithmetic over classical logic. Ruitenburg in [1] axiomatized **BA** and using Kripke model theory, proved that **BA** has *disjunction* and *explicit definability* properties.
We show that **BA** is closed under the Friedman translation and using this translation, it is shown that **BA** is closed under a restricted form of the Markov rule. Moreover, it is also proved that all nodes of a finite Kripke model of **BA** are classical models of $\mathbf{IE}_1^+$, a fragment of Peano arithmetic with Induction restricted to the formulas made up of $\exists$, $\wedge$ and/or $\vee$.

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Tatiana Arrigoni

## What's the problem with $V$?

In the present contribution I'll make some philosophical remarks on $V$, *the* universe of all sets. Since the main business of philosophy is asking questions on matters we find troubling (and giving hints how to answer them in a convincing way), I'm going to begin by considering whether $V$ is a troubling notion or not. Indeed both mathematicians and philosophers seem to have problems with $V$. *Qua* working mathematician one may be puzzled by the fact that $V$, though being usually appealed to as *the* universe of all sets, neither is *de facto* the only one structure satisfying ZFC + large cardinal axioms nor is it the most suitable framework to work within: unfolding the content of $V$ has revealed much more difficult than unfolding the content of, say, $L$. Explaining this by saying that $V$ looks like more natural or that it is more real than other set theoretical universes is no less problematic an explanation than the *explicandum* itself. In fact what do these expressions mean? Is $L$ less real than $V$ and is working in $L$ less natural than working in $V$? On my part, I won't view $V$ as more real or as more natural than other universes. Rather I'll try to explain the fact that $V$ is appealed to as *the* universe of all sets in the practice of set theory by analysing set theorists' shared expectations from their work in $V$. Adopting a *descriptive approach* to set theoretic practice I'll interpret $V$ as a heuristics for doing set theory, coexisting with competing heuristics, which do not mutually exclude but have a bearing on each other.

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MATTHIAS BAAZ

## Axiomatizability of First-order Gödel Logics

joint work with Norbert Preining and Richard Zach

Takeuti and Titani [3] investigated the first-order version of infinite valued Gödel logic with the real interval $[0, 1]$ as its set of truth values. More generally, any closed $V \subseteq [0, 1]$ containing both 0 and 1 may be taken as a set of truth values. Different truth value sets $V$ give rise to different logics. We show that the resulting logic is axiomatizable iff either (1) $V$ is finite; (2) uncountable but 0 is isolated; or (3) uncountable and 0 is in the perfect kernel of $V$. For each finite cardinality, the set of validities is different; and there are two axiomatizable infinite-valued first-order logics corresponding to cases (2) and (3). In all other cases, the resulting logic is not r.e. Similar results are obtained if the logic also contains the $\triangle$ operator ($\triangle(A) = 1$ if $A = 1$ and $= 0$ otherwise). This extends previous results [1, 2].

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SILVIA BARBINA

**Reconstruction of classical geometries from their automorphism group**

Let $V$ be a countably infinite dimensional vector space over a finite field $F$. Then $V$ is $\omega$-categorical, and so are the projective space $\mathrm{PG}(V)$ and the projective symplectic, unitary and orthogonal spaces on $V$. Using a reconstruction method developed by M. Rubin we prove the following result: let $\mathcal{M}$ be one of the spaces above, and let $\mathcal{N}$ be an $\omega$-categorical structure such that $\mathrm{Aut}(\mathcal{M}) \cong \mathrm{Aut}(\mathcal{N})$ as abstract groups. Then $\mathcal{M}$ and $\mathcal{N}$ are bi-intepretable.

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LIBOR BĚHOUNEK

## Extensional set equality over Gödel logic

Gödel predicate logic $G\forall$ is both an important intermediary logic and one of three salient t-norm fuzzy logics. This contribution investigates semantic properties of extensional equality and related notions in set theories over $G\forall$.

*Equality* in an $\{\in\}$-theory over $G\forall$ is a relation $=$ which satisfies the axioms of reflexivity and intersubstitutivity salva veritate, or equivalently, which is an equivalence relation congruent w.r.t. $\in$. It is *extensional* iff it further satisfies the axiom of extensionality $(\forall q)(q \in x \leftrightarrow q \in y) \rightarrow x = y$.

It is well-known [1] that in $G\forall$, an equivalence relation generates a pseudoultrametrics on the universe of discourse. I show further consequences of the congruence and extensionality axioms for membership functions of sets in fuzzy models, i.a. the following

> *Theorem:* Let $=$ be an equivalence left-congruent w.r.t. $\in$. Then the membership function of any set is *uniformly continuous* (1-Lipschitz) and outside its kernel also *locally constant,* w.r.t. the pseudoultrametrics generated by $=$.

Further I discuss the (limited) adequacy of the notion of extensional equality for capturing the graded identity of fuzzy sets over $G\forall$, and the general insufficiency of the expressive power of $G\forall$ for a full-fledged fuzzy set theory.

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Oleg Belegradek

## Collapse under total independence

For a finite relational language $\sigma$, any interpretation of symbols in $\sigma$ as *finite* relations on a set $M$ is called a *state* of $\sigma$ on $M$. A *$\sigma$-query* over $M$ is defined to be any set of states of $\sigma$ on $M$. We call a *$\sigma$-query* $Q$ over $M$ *locally order-generic* with respect to a fixed linear order on $M$ if, for any partial order-preserving map $f$ from $M$ to $M$ and any state $s$ of $\sigma$ on $\mathrm{dom}(f)$, we have $s \in Q$ iff $fs \in Q$.
Let $\mathcal{M}$ be an $L$-structure, where $L \cap \sigma = \emptyset$. Any state $s$ of $\sigma$ on its universe $M$ gives rise to an $(L \cup \sigma)$-expansion of $\mathcal{M}$, and so any sentence in $L \cup \sigma$ defines a $\sigma$-query on $M$. Suppose $\mathcal{M}$ is *infinite*, and $L$ contains $<$ interpreted in $\mathcal{M}$ as a linear order. We say that $\mathcal{M}$ exhibits *generic collapse to order* if, for any $\sigma$, any locally order-generic $\sigma$-query over $M$ definable by a first order sentence in $L \cup \sigma$ is definable by a first order sentence in $\{<\} \cup \sigma$. Clearly, $\mathcal{M}$ exhibits generic collapse to order iff it is so for all models of $\mathrm{Th}(\mathcal{M})$.
A subset $S$ of a structure $\mathcal{N}$ is said to have the *independence property* if there is a formula $\phi(\bar{x}, \bar{y})$ such that for any $n \in \omega$ and $s \subseteq n$ there are families of tuples $\{\bar{a}_i : i \in n\}$ in $S$ and $\{\bar{b}_s : s \subseteq n\}$ in $N$ such that $\mathcal{N} \models \phi(\bar{a}_i, \bar{b}_s)$ iff $i \in s$. In [1] it was proven that $\mathcal{M}$ exhibits generic collapse to order if in $\mathcal{M}$ there is an infinite definable set without the independence property, and asked whether the converse is true. We answer the question in the negative.
A complete theory $T$ is said to have the *total independence property* if for any infinite definable subset $S$ of a monster model of $T$ there are a definable binary relation $R$ on $S$ and families $\{a_i : i \in \omega\}$, $\{b_s : s \subseteq \omega\}$ of elements of $S$ such that $(a_i, b_s) \in R$ iff $i \in s$.

Theorem. *Let $V$ be an ordered vector space over an ordered skew-field. Then any expansion of $V$ by bounded relations exhibits generic collapse to order, but there exists an expansion of $V$ by finitely many bounded relations such that the theory of the expansion has the total independence property.*

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Luca Bellotti

## The problem of consistency: *ZF* and large cardinals

My aim is to discuss, on the basis of a historical survey, the question of the consistency of ZF set theory and of its large cardinal extensions. First, I explain the reasons why in the case of set theory neither model–theoretic nor proof–theoretic methods seem sufficient to come to grips with the problem of consistency. Then, I recall how set–theorists have dealt with the problem by means of large cardinals and inner models, with a reasonably confident attitude about consistency. Finally, I hint at finitary versions of Gödel sentence constructions (due to W. H. Woodin) which could allow a comparison between arithmetic and the extensions of set theory with respect to our knowledge of their consistency. I maintain that the main philosophical interest of the problem lies in the relationship between intuition and formalization.

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Keywords: set theory, consistency, large cardinals.

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## Polynomial Time Programming Language

A programming language (similar to Pascal) is proposed in which only all polynomial time computable functions can be programmed. More precisely: any program in the proposed language, computing a function of words in some finite alphabet (with word values), can be executed by the standard interpreting Turing machine in the time bounded by some polynomial of the length of the input word. Any polynomial time computable function can be also programmed in the proposed language. It is allowed to use natural numbers and functions in the proposed language. Arguments and values of the functions may be not only numbers but functions also. All arguments must be precisely specified (including arguments of functional arguments and so on). Recursion is not allowed. Values ranges must be specified for all variables. But range frames may be computed while entering corresponding routines. It means that range frames may be defined by expressions. Basic operations in expressions are addition, positive subtraction, multiplication, integral division. Defined functions may also be used in expressions. Variable arrays are allowed (with computed indices ranges). Arrays can also be function arguments and values. Array index range may depend on previous arguments.
The simpliest language statement is an assignment operator. The function value is defined by the expression at the end of its body. Conditional statement may also be used. Conditions are comparisons of numeric expressions ("greater" or "equal"). Loops are allowed to be only "for"-type without changing the loop variable within the loop body. Words of a finite alphabet are simulated by arrays.

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Stefano Berardi

## Classical Arithmetic as solution of a Maximum problem

Call $L$ the language of arithmetic. Denote by $\forall\exists_n$ the set of closed formulas $\forall x.\exists y.P(x,y)$ of $L$, with $\deg(P) = n$. We order intuitionistic theories over $L$ as follows. We first compare their sets of $\forall\exists_0$-theorems. If they are the same, we compare their sets of $\forall\exists_1$-theorems. If they are the same, we compare their sets of $\forall\exists_2$-theorems. And so forth.
Call $\leqslant_{\forall\exists}$ the resulting ordering on intuitionistic theories. We prove the following result, in a purely intuitionistic way. There is a maximum, w.r.t. $\leqslant_{\forall\exists}$, among the intuitionistic theories extending Heyting Arithmetic `HA` which are *informative* (whose $\forall\exists_0$-theorems are all true). This maximum is exactly `LCM`, Intuitionistic Limit Interpretation of Classical Arithmetic `PA` (see [1],[2], [3]). This result characterizes `LCM` among the many existing intutionistic Interpretation of `PA`.
We may restate our result as follows. We want to extend `HA` with axioms which are specification of maps over Natural Numbers. We want to choose an informative set of axioms. Whenever two axioms are inconsistent each other, we prefer adding the axiom of lower degree. Then there is a maximum extension we may obtain in this way. This extension includes `PA`.

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Josef Berger

## Unique Existence Theorems in Constructive Analysis

We are interested in unique existence theorems in the framework of Bishop's constructive mathematics.

In [1] we investigate under which conditions a uniformly continuous mapping on a compact metric space has a fixed point. It turns out that a natural unique existence theorem is equivalent to Brouwer's fan theorem.

This nicely fits into a series of recent results:

In [2] we prove that the fan theorem is equivalent to the following statement:

*Each uniformly continuous function on a compact metric space with at most one minimum attains its minimum.*

In [3] Bridges introduces a new and constructive relevant notion of sequential compactness, and shows that the fan theorem is equivalent to the statement:

*Each compact metric space is sequentially compact.*

Hence certain **unique existence theorems** are equivalent to the fan theorem. We (including the authors in the references below) want to know why this is the case. Is there even a meta-theorem which classifies all unique existence theorems that are equivalent to the fan theorem?

This question is very much in the spirit of constructive reverse mathematics as proposed by Ishihara in [4]: given a statement $A$ which is valid classically, to look for a general principle $P$ such that $A$ and $P$ are constructively equivalent.

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MARTA BILKOVA

**A computational view of intuitionistic propositional proofs**

We shall present a possible view of intuitionistic propositional sequents as (in special cases polynomial time) computations. We start with the $\{\wedge, \vee, A \rightarrow H\}$ fragment of IPC with $H$ a Harrop formula.
We consider the Disjunctive form $A^D$ of a formula $A$ to be the disjunction of conjunctions of atoms and implications equivalent to $A$. Let the length $|A|$ of a formula $A$ be the number of disjuncts in its disjunctive form and $A_i$ be the $i$-th disjunct of $A^D$.
Consider a sequent $\mathsf{S} : \Gamma \Rightarrow A$ and let $f^{\mathsf{S}} : \{0, \ldots, |\Gamma|-1\} \mapsto \{0, \ldots, |A|-1\}$ be a function such that $\Gamma_i \rightarrow A_{f^{\mathsf{S}}}(i)$ is a tautology of IPC.
For $\mathsf{S}$ a provable sequent there is such a function.
We prove the following: There is a polynomial time function $F$ such that for every proof $\pi$ of a sequent $\mathsf{S}$, $F(\pi)$ is a formula expressing a function $f^{\mathsf{S}}$. Moreover $f^{\mathsf{S}}$ is also a polynomial time function.
We can show a possible way how to deal with more complex implications and a connection with disjunction property and feasible interpolation.

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STEFAN BOLD

## AD and Descriptions for cardinals below $\Theta$

We work throughout in the theory $\mathsf{ZF} + \mathsf{AD} + \mathsf{DC}$. In the mid-80s Steve Jackson solved the fifth Victoria Delfino problem by computing the values of the projective ordinals $\boldsymbol{\delta}^1_n$. A key part of this analysis was the concept of descriptions, finitary objects that "described" how to build ordinals less than a projective ordinal. In 1990 Steve Jackson and Farid T.Khafizov showed that below $\boldsymbol{\delta}^1_5$ the descriptions really correspond to the cardinals less than $\boldsymbol{\delta}^1_5$. An application for this finer analysis was to compute the cofinalities of all cardinals below $\aleph_{\omega^{\omega^\omega}+1} = \boldsymbol{\delta}^1_5$. In this talk, we report on work in progress concerning the generalization of the Jackson-Khafizov analysis to the higher projective ordinals.

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Roger Bosch

## Definably large cardinals

By a definably large cardinal $\kappa$ we mean an inaccessible cardinal which has some other property of the classical large cardinals restricted to first order definable subsets of $V_\kappa$. For instance, a $\kappa$ is a $\mathbf{\Sigma}_n$-Mahlo cardinal iff it is inaccessible and every club on $\kappa$ which is definable by means of a $\Sigma_n$ formula (with parameters) in $V_\kappa$ contains an inaccessible cardinal. In [1], [2] and [3] some of these cardinals have been used to prove several consistency results about properties of definable sets. We plan study some types of these cardinals in order to show several theorems about their place in the hierarchy of large cardinals and about their own hierarchy.

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Andrey I. Bovykin

## Several proofs of $PA$-unprovability

We shall sketch the proofs of several new combinatorial statements independent of Peano Arithmetic. All of them are proved model-theoretically using different variations of the method of indiscernibles. Related results of A.Weiermann and G.Lee will be discussed.

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Jens Brage

## A natural interpretation of classical proofs

We extend the double negation interpretation from formulas to derivations. The resulting interpretation is compositional and respects the most obvious conversion rules of the Gentzen sequent calculus for classical predicate logic. It furthermore induce a notion of classical proof similar to that of intuitionistic logic. The interpretation agree with the Kolmogorov interpretation of formulas not containing implication.

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Riccardo Bruni

## A Note on Gödel's Philosophy of Mathematics in the Light of His *Nachlass*

The problem of giving a coherent unifying picture for all aspects of Gödel's philosophy of mathematics, still appears as an appealing task. The present research, is intended as a contribution to this direction of work.

We start by shortly summarizing Gödel's philosophy of mathematics, focusing the attention on the crucial role of the act of understanding abstract concepts. We then offer a possible explanation of Gödel's position, trying to make clear its relation with Gödel's re on "axioms of infinity" as possibile ways to significantly extend our set-theoretical knowledge. In other words, we try to give an answer to the following question: Why did Gödel insist on similar principles, while stressing their unusefulness to solve the problem (decidability of Cantor's Continuum Hypothesis) which primarily indicated the need for the above mentioned extension?

Two remarks turn out to be of special importance: (i) these axioms stem from the very concept (the iterative conception of set) which is at the base of a "truly mathematical" theory of the informal notion of "aggregate"; (ii) they make decidable otherwise undecidable Diophantine propositions. We give an analysis of how Gödel argumented in favor of these reasons, thus emphasizing, in particular, a relation with Gödel's re on the significance of the incompleteness theorems.

We make an essential use of only recently published material coming from Gödel's *Nachlass*, which helps us to recast some of his known papers in an hopefully new perspective.

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DUMITRU BUŞNEAG

**Localization for some algebras of fuzzy logic**

joint work with Florentina Chirteş and Dana Piciu

Let $A$ be an MV (BL, pseudo MV, pseudo BL or n-valued Lukasiewicz-Moisil) algebra and $\mathcal{F}$ a topology on $A$. The scope of this contributed paper is to define the localization algebra $A_{\mathcal{F}}$ of $A$ relative to $\mathcal{F}$ (as in the case of rings or bounded distributive lattices) and to describe $A_{\mathcal{F}}$ in some particular cases.

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Anahit A. Chubaryan

## On the proofs complexity in the Resolution systems of Intuitionistic and Minimal propositional logic

joint work with Armine A. Chubaryan

It is well-known, that there is some hierarchy of the proof systems of Classical propositional logic (CPL) under the $p$-simulation relation. It is interesting to investigate this relation amongst the proof systems of Intuitionistic propositional logic (JPL) and also amongst the proof systems of Minimal (Joganson's) propositional logic (MPL). This problem is not trivial because some of the systems of CPL have the speed-up over the analogous systems of JPL and MPL.
The known systems of JPL and MPL are: Hilbert's style systems (HJP, HMP), sequent systems with cut rule (SJP, SMP) and cut-free (SJP$^-$, SMP$^-$), resolution and natural systems of JPL (RJP and NJP), described by Mints, and analogous systems of MPL (RMP and NMP), described by us.
Earlier it was proved by An. Chubaryan, that the systems HJP and HMP with substitution have the speed-up (by lines) over the analogous systems without substitution. It isn't difficult to prove, that the systems HJP, SJP, NJP (HMP, SMP, NMP) are $p$-equivalent and they have the speed-up over the systems RJP, SJP$^-$ (RMP, SMP$^-$ accordingly).
Now we prove that
1) the systems RJP and SJP$^-$ (RMP and SMP$^-$) are $p$-equivalent;
2) for sufficiently large $n$ and every $t$ $\left(1 \leqslant t \leqslant \left[\frac{n}{2} log_n 2\right]\right)$ there are tautologies $\varphi_n^t$ of the length $n$, the proofs complexities (size, lines) of which in above four systems are $\theta\left(n^t\right)$.
Earlier, using the notion of $\varphi$-determinative conjuction for any tautology $\varphi$, we obtained the similar results for the analogous systems of CPL. Some modification of this notion is acceptable also for intuitionistic and minimal logic.

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P. Cintula

**Towards Universal Fuzzy Logic**

Many-valued logics constitute an important class of non-classical logics. During the previous century many logical systems of many-valued logics were developed. The reasons for using more than two values were mainly philosophical and linguistical. The new impulse to this area of logic was given by fuzzy logic.

Originally, fuzzy logic was a label for a class of engineering tools rather than mathematical or logical system. However, at the end of the previous century the *mathematical fuzzy logic* was introduced and then developed by logicians like Hájek, Montagna or Mundici into the well-defined logical system. In fact wide class of logical systems was introduced.

The goal of this contribution is to study systematically this class of logics from an abstract point of view. As a starting point we examine Hájek's concept of a "logic of comparative degrees of truth". Using this concept we introduce the *minimal* fuzzy logic and then develop a theory of its extensions.

It turns out that some well-known logics are covered by this approach, namely Intuitionistic and many substructural logics. This is caused by the connection between our theory and Abstract algebraic logic. However, we deal with a very special (narrow) class of logics (from the AAL point of view) and we classify logics in quite different way.

The main contribution of this work is the first-order part of Abstract fuzzy logic. We present a uniform way of defining predicate logic for given propositional logic. We prove several theorems (completeness, Skolem terms introduction, etc.) in this very general setting.

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CAMILLO COSTANTINI

**Consistent examples about extensions of continuous functions**

joint work with Dmitri Shakhmatov

In the paper *Extensions of functions which preserve the continuity on the original domain* (Topology and its Applications, **103** (2000), 131–153), C. Costantini and A. Marcone proved that a topological space $X$ is (hereditarily) normal and hereditarily extremally disconnected if and only if for every compact metrizable space $Y$ and every continuous function $f$ from a subspace of $X$ to $Y$, there is a continuous $\tilde{f}\colon X \to Y$ which extends $f$. A natural question in this vein is whether in the above statement we may drop the hypothesis of metrizability on $Y$ (leaving unchanged the assumption of compactness).
In the present work, we first show that a space drafted by Kenneth Kunen in the Notices of the American Mathematical Society (1977) may be suitably modified to give a ZFC counterexample to such a question. We also point out that from another result, illustrated by F.D. Tall in the Handbook of Set-theoretic Topology, it follows that under the assumption $2^\omega = 2^{\omega_1}$ the desired space may be obtained to be separable.
Then we show how it is consistent that a very large variety of such examples exist. Actually, using forcing, it is possible to find a model of ZFC where every separable, first countable space of cardinality $\omega_1$ admits a stronger topology $\sigma$, such that $(X,\sigma)$ is a separable, hereditarily normal, hereditarily extremally disconnected space, and there exists a continuous function $f$ from a subspace of $(X,\sigma)$ to $\beta\omega_1$ which cannot be continuously extended to the whole of $(X,\sigma)$. The key point for the proof are certain combinatorial properties that are valid in such a model, and which may turn out to be of some independent interest.
Finally, working in the opposite direction, we show that there exists in ZFC a separable (nonempty) space $X$, without isolated points, such that every continuous function $f$ from a subspace of $X$ to a compact space $Y$ can be continuously extended to the whole of $X$.
(All spaces are meant to be Hausdorff).

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LAURA CROSILLA

**On constructing completions**

joint work with Hajime Ishihara and Peter Schuster

Constructive Zermelo Fraenkel set theory (CZF, P. Aczel 1978) is a subsystem of ZF which is based on intuitionistic logic. It is characterised by allowing only restricted separation and refraining from powerset, which is replaced by a scheme of subset collection. These constraints are introduced to adhere to a notion of predicativity.
Subset collection probably amounts to the most intricate principle of CZF. On the basis of the other axioms of CZF, it may be rephrased as a single axiom, called fullness. This states that given two sets $a$ and $b$, there exists a set $c$ of total relations between $a$ and $b$ such that for every total relation between $a$ and $b$ there is a subrelation of it belonging to $c$. Fullness has been utilised by P. Aczel to show that the constructive Dedekind reals do form a set in the context of CZF. We shall formulate a consequence of fullness, which we call the principle of refinement. This sufficies to show, for example, that the completion of an arbitrary ordering is a set, thus allowing us to generalise Aczel's achievement for Dedekind reals.

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PAOLA D'AQUINO

**Quadratic reciprocity law in weak fragments of $PA$**

joint work with Angus Macintyre

There are around 150 proofs of Gauss classical result on quadratic reciprocity law. They all seem to use functions of exponential growth or counting arguments which are not available in $I\Delta_0$. We consider Gauss second proof of quadratic reciprocity law which uses quadratic forms. We develop the theory of quadratic forms over $I\Delta_0 + \Omega_1$, and we study the connections with Pell equations. Some cases of quadratic reciprocity law are proved.

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L. Darniere

## Model-completions of scaled lattices

The theory of boolean algebras is well known to admit as a model-completion the theory of dense boolean algebras. There is a wide class of lattices in which the Krull-dimension of the spectrum (that is the Stone space of prime filters) is definable in a natural way, we call it the class of scaled lattices. Boolean algebras, for example, are precisely scaled lattices of dimension zero (by which we mean scaled lattices whose spectrum has Krull-dimension zero). Our motivating example of non-zero dimensional scaled lattice is the lattice of all closed definable subsets of the power set $k^N$ with $k$ an algebraically closed, real closed or $p$-adically closed field. It is remarkable that the dimension of lattice coincides in these cases with the usual geometric dimension as we will check it. We prove that the theory of scaled lattices of arbitrary dimension N admits a model-completion and give an explicit axiomatization of it, which boils down to the usual one (density) in the zero dimensional case.

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Jairo José da Silva

## On the principle of excluded middle

The principle of excluded middle (PEM) is one of the basic principles of classical logic, but its universal validity is sometimes disputed. I will consider here the form of PEM known as the principle of bivalence: any judgment is either true or false, tertium non datur. The validity of this principle depends on certain presuppositions (as has already been largely acknowledged), but I claim that these presuppositions are not of a mathematical or metaphysical nature, being, on the contrary, best understood as regulative transcendental principles. In fact the validity of PEM depends on the presupposition that any proper assertion is in principle decidable, but I claim that this does not mean that there is somewhere, in any sense of existence, a decision procedure that, given a proper assertion, can determine its truth-value. In order to show this I will provide in this talk a clarification of what decidability in principle means. PEM applies exclusively to proper judgments, which I characterize as those that have both syntactic and semantic senses, i.e. those that conform to both syntactic and semantic rules. The latter, in particular, tell us what to expect as contents of direct (evidential) experiences; they determine, from the point of view of our limited past experiences, the possible content of the whole domain of our future experiences (and this gives them their transcendental character). This means that proper judgments denote situations that can be experienced. They are then, we say, decidable in principle and have (and this is an equivalent formulation), an intrinsic truth-value (either true or false), regardless of whether we are now, or will ever be in a position to actually decide which.

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DEJAN DELIĆ

## Finite Decidability of Locally Finite Varieties

One of the outstanding problems in the contemporary general algebra has been the quest for the description of locally finite equational theories (i.e. locally finite classes of algebras defined via equalities of terms in a given language) whose first-order theory is decidable.
A class of algebras is considered to be 'badly structured' if a structurally rich class of structures, e.g. the class of all (finite) graphs, can be interpreted in it using a scheme of first-order formula with parameters. In universal algebra, this notion is intimately related to the one of decidability, and, in particular, the decidability of the subclass of all finite members. The techniques of tame congruence theory have proved to be the principal tool in this line of investigation. In this talk, we shall outline the structural features of algebras in finitely decidable locally finite varieties.

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Alexander Denisov

## Successful Modification of the Nuclear Theory

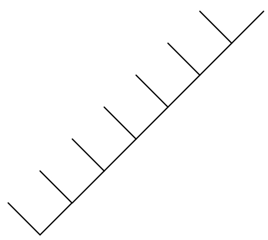
For axiomatic of the nuclear theory $T_{\mathrm{nuc}}$ see [1]. The most important property is

*Proposition.* A lot of the problems in the theory of constructive models can be solved by building of necessary theory or model with a help of modification $T_{\mathrm{nuc}}$ or $\mathcal{M}_{\mathrm{nuc}}$.

Among origins of that proposition are the following elementary tasks which are solvable with a help of the nuclear theory:

1. Construct a nonconsistent theory.
2. Construct a theory which admits quantifier elimination.
3. Construct a complete theory.
4. Construct a decidable theory.
5. Construct a non $\alpha$-categorical theory for all $\alpha \geqslant \omega$.

The structural diagram of $T_{\mathrm{nuc}}$ consists of decreasing sequence of three circles with correlated symbols $P_0, P_1, P_2$, dots and inner punctuated circle. But its variant



solves also very successively. In particular it is good for the task

6. Construct a decidable theory with nonrecursive prime and saturated models [2].

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Mauro Di Nasso

## Different paths to nonstandard analysis

joint work with Marco Forti

We briefly introduce two different "paths" to nonstandard analysis. In algebraic terms, we characterize the hyperreals $^*\mathbb{R}$ as those fields that are homomorphic images of *composable rings* of real functions. In terms of ultrafilters, we obtain sets of hyperintegers $^*\mathbb{Z}$ as suitable invariant subspaces of the Stone-Čech compactification $\beta\mathbb{Z}$, where a structure of ring is obtained by considering natural sum and product operations. The existence of such subspaces is equivalent to the existence of special ultrafilters called Hausdorff ultrafilters, which have been recently shown to be independent of ZFC.

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NATASHA DOBRINEN

**Almost everywhere domination**

joint work with Stephen G. Simpson

Let $2^\omega$ (the space of infinite sequences of 0's and 1's) be endowed with the usual "fair coin" measure. We say that a Turing degree $\mathbf{d}$ is *almost everywhere dominating* if for almost all $X$ in $2^\omega$ there exists a function of degree $\mathbf{d}$ which eventually dominates all functions recursive in $X$. We discuss the problem of characterizing the almost everywhere dominating Turing degrees and present some partial results. The motivation for this work comes from a well-known theorem about measure algebras in set theory.

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Christoph Duchhardt

## Reflection vs Thinning

We show how to use thinning operations (which were proposed by M.Moellerfeld) in order to proof-theoretically analyze systems of set theory based on reflection rules.

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PHILIP EHRLICH

## The Rise of non-Archimedean Mathematics and the Roots of a Misconception

In his paper *Recent Work On The Principles of Mathematics*, which appeared in 1901, Bertrand Russell reported that the three central problems of traditional mathematical philosophy—the nature of the infinite, the nature of the infinitesimal, and the nature of the continuum—had all been "completely solved." According to Russell, the structure of the infinite and the continuum were completely revealed by Cantor and Dedekind, and the concept of an infinitesimal had been found to be incoherent and was "banish[ed] from mathematics" through the work of Weierstrass and others. These themes were further developed in both the 1903 and 1937 editions of Russell's *The Principles of Mathematics*, the works which perhaps more than any other helped to promulgate these ideas among historians and philosophers, of mathematics. In the latter works, however, the banishment of infinitesimals that Russell spoke of in 1901 was given an apparent theoretical urgency. No longer was it simply that "nobody could discover what the infinitely little might be," but rather, according to Russell, the kinds of infinitesimals that had been of principal interest to mathematicians were shown to be either "mathematical fictions" whose existence would imply a contradiction or, outright "self-contradictory," as in the case of an infinitesimal line segment. In support of these contentions Russell could cite no less an authority than Georg Cantor, the founder of the theory of infinite sets.
In a the present talk, which provides a synopsis of [1], attention will be drawn to just how misleading the just-cited views of Russell are vis-à-vis late nineteenth-century mathematics and to just how mischievous those views of Russell have been.

[1] P. Ehrlich, *The Rise of non-Archimedean Mathematics and the Roots of a Misconception I: the Emergence of non-Archimedean Grössensysteme*, **Archive for History of Exact Sciences** (forthcoming).

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Fredrik Engström

## Notions of resplendency for logics stronger than first-order logic

A model $M$ in a recursive language $L$ is *resplendent* if for any recursive theory $T$ in a recursive extension of $L(\bar{a})$, where $\bar{a} \in M^{<\omega}$, such that $\mathrm{Th}(M, \bar{a}) + T$ is consistent there is an expansion of $M$ satisfying $T$.
We will present some similar notions of expandability for logics stronger than ordinary first-order logic. The main notion concerns first-order logic extended with the ability to express the property of omitting a type (which is an $L_{\omega_1\omega}$ sentence). Models which satisfy this notion of expandability are called *transcendent*.
As a special case we get a resplendency notion for first-order logic with a standard predicate expressing when an element is a (standard) natural number. We characterise all such countable models of arithmetic in terms of their standard systems.

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HORACIO FAAS

**Visual and Heterogeneous Inferences**

In the last two last decades of twentieth century there have been a process to rescue visualization in the mathematical procedures, nowadays continued, although that position is still considered as heretical by most of the specialists. The reasons of the rejection lie on the mistakes that can contribute our sensible intuitions to us, specially vision in this case. For that reason the figures and the diagrams are relegated to an aid for the understanding, but never it is allowed that they play an active role in the demonstrations. Some of the so called "mathematical monsters" have paid the field to maintain that position of rejection. The reliable inferences are solely those formalized in a language. Nevertheless, it cannot be refused that in daily life we make inferences without appealing to linguistic intermediation. As said above, visualization is coming back accompanied sometimes by rigourous presentations to arrive, in some cases, to accept it like genuine part of demonstrations. There exist now systems of heterogenous inference, that is to say, systems that allow inferences between two different representation systems: one of them diagrammatic and another linguistic one. In order to do it, it is necessary to consider possible extensions of a situation. Since a situation offers partial information, its extension enriches it. This new situation implies the first one; it is able to interpret more syntactic characteristics than the one that operates like premise, but it agrees with this one in which it already possess. In this paper I talk about heterogenous inference and some well known cases of visual demonstration, applying to them this notion of extension and I show how the misuse of it influences the production of the apparent paradoxes to which the mentioned mathematical monsters give rise. Particularly, it will be shown how the graphical presentation of the continuous and nondifferentiable functions elaborated from Weierstrass' initial one do not respect the restrictions that the mentioned notion of extension of a diagram imposes.

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JOSEP MARIA FONT

**New forms of the Deduction Theorem and Modus Ponens**

The paper [**1**] studies, with techniques of Abstract Algebraic Logic, the effects of putting a bound on the number of "side formulas" in the Deduction Theorem, viewed as the Gentzen-style rule:

$$\text{(DT)} \qquad \frac{\Gamma, \varphi \rhd \psi}{\Gamma \rhd \varphi \to \psi},$$

where $\rhd$ is the separator of the two sides of a sequent. A denumerable collection of Gentzen systems $\mathfrak{G}_n$ is defined, with structural rules, the Modus Ponens rule (MP) $\varphi, \varphi \to \psi \rhd \psi$, and the rule (DT$n$) equal to (DT) but with $|\Gamma| \leqslant n, (n \neq 0)$. We study them and the sentential logics they define; we determine their algebraic models and the relationships between them, and classify them in the *protoalgebraic hierarchy*, according to standard criteria of Abstract Algebraic Logic.
Proof-theoretically all the $\mathfrak{G}_n$ are different. For each $n \geqslant 2$ the logic defined by $\mathfrak{G}_n$ is $\mathcal{IPL}_{\to}$, the implicative fragment of intuitionistic logic. This yields new abstract characterizations of $\mathcal{IPL}_{\to}$, in the TARSKI style. The logic $\mathcal{G}_1$ defined by $\mathfrak{G}_1$ is weaker than $\mathcal{IPL}_{\to}$, is selfextensional and protoalgebraic, but is neither equivalential nor weakly algebraizable, a rare situation where very few examples were hitherto known.
We also consider the following strengthenings of Modus Ponens:

$$\text{(MP}n\text{)} \quad \left.\begin{array}{l} \varphi_1 \to \big(\cdots \to (\varphi_n \to \varphi) \cdots\big) \\ \varphi_1 \to \big(\cdots \to \big(\varphi_n \to (\varphi \to \psi)\big) \cdots\big) \end{array}\right\} \rhd \varphi_1 \to \big(\cdots \to (\varphi_n \to \psi) \cdots\big)$$

We show that $\mathcal{IPL}_{\to}$ is axiomatized by axiom (K) $\varphi \to (\psi \to \varphi)$, rule (MP) and one rule (MP$n$) for a single $n \geqslant 3$. The logic $\mathcal{H}_1$ defined by (K) , (MP) and (MP2) is an algebraizable non-selfextensional companion of $\mathcal{G}_1$, but is still weaker than $\mathcal{IPL}_{\to}$. Its equivalent algebraic semantics is the class $\mathsf{QH}$ of *quasi-Hilbert algebras*, a quasi-variety of implicative algebras strictly larger than the variety of *Hilbert algebras*.
The algebraic counterpart of $\mathcal{G}_1$ is the variety generated by $\mathsf{QH}$. The problem of whether $\mathsf{QH}$ is actually a variety or a proper quasi-variety remains open.

[**1**] Bou, F., Font, J. M., García Lapresta, J. L., On weakening the Deduction Theorem and strengthening Modus Ponens, *MLQ* 50 (2004) 303–324.

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ANDREY FROLOV

## Computable copies of distributive lattices with relative complements and linear orderings

Feiner showed ([1]) that a $\Delta_2^0$ Boolean algebra need not have a computable copy. Knight and Stob ([2]) showed that every $low_4$ Boolean algebra is isomorphic to a computable one. In my talk I'll show that every $low_4$ distributive lattice with relative complements has a computable copy. Also I talk about spectrume of linear ordering of several order types.

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NIKOLAOS GALATOS

**Translations in substructural logic**

joint work with Hiroakira Ono

Substructural logics are logics that, when formulated in Gentzen systems, lack the substructural rules of exchange, weakening and contraction. One of the most prominent such systems is the full Lambek calculus, **FL**. We discribe a Hilbert system **HL** that is equivalent to **FL** (the two systems are mutually translatable), provide an algebraic proof of the cut elimination of **FL** and show the strong separation property for **HL**. Moreover, we note that **HL** is algebraizable and the equivalent algebraic semantics is the variety $\mathcal{RL}$ of residuated lattices (the deducibility relation of **HL** and the quasi-equational theory of $\mathcal{RL}$ are mutually translatable), we provide an axiomatization of the classes of subreducts of $\mathcal{RL}$ and indicate how to obtain an embedding of a partial subalgebra of a residuated lattice into a new residuated lattice that preserves finiteness in certain cases.

Finally, we show that the general setting of substructural logic is a natural context in which various translations - including Glivenko, Kolmogorov and Gödel translations - between logics can be formulated and proven.

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Çiğdem Gencer

## Unification and Passive Inference Rules for the Modal Logic $\boldsymbol{K}4$

A complete syntactic description of all formulas which are non-unifiable in wide classes of modal logics is given and it is shown that in any modal logic over $S4$ there is a finite basis for passive rules in [1]. Using the same technique, it is shown that in any modal logic over $D4$ there is a finite basis for passive rules in [2]. In this paper, we study passive inference rules in the modal logic $K4$.

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Paolo Gentilini

## Critical Deduction Chains and Inferential Models for Type Theory

joint work with Maurizio Martelli

A sequent version $\mathbf{LK}_\omega$ of full higher order logic is considered, whose rules are similar to that of the higher order Abstract Logic Programming Languages introduced in [4]. $\mathbf{LK}_\omega$ is a sequent presentation of a Church-style type theory, (see e.g. [1]). An *inferential semantics* for $\mathbf{LK}_\omega$ is provided, starting from the fact that the *Comprehension-rules* $\exists R$, $\forall L$ of $\mathbf{LK}_\omega$, expressing the Comprehension Axiom play a central role in the $\mathbf{LK}_\omega$ proofs. Inferential semantics also develops some formal tools already introduced, for different purposes, in [2] and [3]. In a higher order proof-tree P, sequences of formulas having a deep inferential meaning can be selected, which express the action of Comprehension-rules in the deduction, that are called *Comprehension Abstract Deduction Structures (Comp-ADS)*. This allows to define an *Inferential Algebra* of proof-trees $Inf\text{-}\mathcal{A} \equiv \langle \{{}^{\mathrm{Kr}}\mathbf{D}_\alpha\}_{\alpha \in types}, \Rightarrow, * \rangle$ based on the *critical chains ADS*, having two binary operations, the *abstraction* $\Rightarrow$ and the *contraction. Inf-A* provides the domains of the inferential semantics. Soundness and completeness of $\mathbf{LK}_\omega$ w.r.t. the inferential algebra *Inf-A* are proven.

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AMÉLIE GHEERBRANT

**Recursive complexity of L-modal logic**

joint work with Marcin Mostowski

Hintikka in [1] claims that the tautology problem for first order modal logic with L–semantics is equivalent to that of second order logic (SO). His argument is based on some intuitive explanations without clear mathematical definition. As a matter of fact in his proof he uses an arbitrary combination of K– and L–semantics. Following his intuitive explanations we define L–modal logic as a first order modal logic with the same set of formulae as usually and interpreted in first order structures by the same inductive conditions as in the first order case, and additionally
$M \models \Box\varphi$ iff for each model $M'$ with the same universe as that of M, $M' \models \varphi$.

**Theorem 1.** *Let $T$ be the set of all tautologies of L-modal logic, then the recursive degree of $T$ is greater than both $\Sigma_1$ and $\Pi_1$ but less or equal to $\Sigma_2$.*

Therefore $T$ is much simpler than SO, nevertheless it is not axiomatisable.

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MATTHEW B. GIORGI

**A High Noncuppable Enumeration Degree**

joint work with Andrea Sorbi

The concept of enumeration reducibility is that of relative computability between sets of natural numbers, where a set $A$ is enumeration reducible to a set $B$ if and only if there is a procedure that uniformly provides an enumeration of $A$ when given any enumeration of $B$. We formalise this concept via the notion of an *enumeration operator* (or simply *e-operator*) which is a mapping $\Phi\colon 2^\omega \to 2^\omega$ for which there exists a c.e. set $W$ such that, for each $X \subseteq \omega$,

$$\Phi^X = \{x \mid (\exists u)[\langle x, u\rangle \in W \wedge F_u \subset X\,]\},$$

where $F_u$ is the finite set with canonical index $u$. We say that a set $A$ is *enumeration reducible* (or simply *e-reducible*) to a set $B$, written symbolically as $A \leqslant_e B$, if and only if $A = \Phi^B$ for some e-operator $\Phi$. We denote by $\equiv_e$ the equivalence relation generated by the preordering relation $\leqslant_e$ and $\deg_e(X)$ denotes the equivalence class (or the *e-degree*) of $X$. The partially ordered structure of the e-degrees $\mathfrak{D}_e = \langle \mathcal{D}_e, \leqslant\rangle$ is an upper semilattice with least element $\mathbf{0}_e$. The $\Sigma^0_2$ e-degrees are precisely the e-degrees $\leqslant \mathbf{0}'_e$. One can define a jump operation ($'$) on the e-degrees and therefore introduce the notions of a *low* e-degree (i.e. an element of the class $\mathbf{L}_1 = \{\mathbf{a} \leqslant \mathbf{0}'_e \mid \mathbf{a}' = \mathbf{0}'_e\}$) and that of a *high* e-degree (i.e. an element of $\mathbf{H}_1 = \{\mathbf{a} \leqslant \mathbf{0}'_e \mid \mathbf{a}' = \mathbf{0}''_e\}$). A $\Sigma^0_2$ e-degree $\mathbf{a}$ is *cuppable* if there exists a $\Sigma^0_2$ e-degree $\mathbf{b} < \mathbf{0}'_e$ such that $\mathbf{a} \cup \mathbf{b} = \mathbf{0}'_e$. And naturally a $\Sigma^0_2$ e-degree which is not cuppable we call *noncuppable*. We demonstrate the existence of a high noncuppable $\Sigma^0_2$ e-degree. Thus showing that there is a high e-degree $\mathbf{h}$ such that all nonzero e-degrees $\mathbf{a} \leqslant \mathbf{h}$ are properly $\Sigma^0_2$.

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Mariusz Grech

## The automorphisms of the lattice of equational theories of commutative semigroups

The first nontrivial automorphism of the lattice of equational theories of commutative semigroups has been found by A. Kisielewicz in [1], showing that not every theory is definable in this lattice. R. McKenzie has generalized the example into an infinite family of the automorphism of order 2. We generalize this result.

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NICOLAS GUZY

**Topological differential structures**

joint work with Françoise Point

We consider first-order theories of topological fields admitting a model-completion and their expansion to differential fields (requiring no interaction between the derivation and the other primitives of the language). We give a criterion under which the expansion still admits a model-completion and provide an axiomatization of it. It generalizes previous results due to M. Singer [5] for ordered differential fields and of C. Michaux [3] for valued differential fields. It differs from the recent results of M. Tressl [6] in two respects, on one hand we are only dealing with a single derivation, on the other hand we are not restricting ourselves to definable expansions of the ring language, taking advantage of our topological context. Our criterion requires a topological assumption which is shown equivalent to the notion of large fields used in [6]. Let us notice that the Pierce-Pillay style axiomatizations of these theories have been obtained in [2].

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Kai Hauser

## The Philosophical Roots of Cantorian Set Theory

I explore systematic connections between the philosophical foundations of Cantorian set theory and various philosophical traditions. Whereas it is difficult to judge whether any of these connections had a truly formative influence on Cantor's mathematical thought, some of them are vindicated by modern developments in axiomatic set theory.

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CHRISTOPH HEINATSCH

**The determinacy strength of $\mathbf{\Pi^1_2}$-comprehension**

joint work with Michael Möllerfeld

Determinacy axioms state the existence of winning strategies for infinite two player games played on the natural numbers.
We show that a base theory enriched by a certain scheme of determinacy axioms is proof-theoretically equivalent to $\Pi^1_2$-comprehension.

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## Csaba Henk

### A Qualitative Approach to the Church-Turing Thesis

In this talk we will discuss a model-theoretic equivalent of the famous Church-Turing Thesis, according to which a function should be considered as effectively computable if and only if it can be computed by means of a Turing Machine. This rewording of the Thesis can serve as a base for a new kind of meta-mathematical verification of it (succintly: a relation is effectively decidable iff can be defined in a way such that the fact whether a given element fulfils the definition depends only on finitely many atomic relations).
As a "by-product" of the above theory we will gain an interesting intensional perspective to the Zermelo-Fraenkel set theory.

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James Hirschorn

**On iterated (proper) forcing without adding reals**

We discuss a method for obtaining new models of CH.

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ALBERT HOOGEWIJS

**Formath: Higher-order logic revised**

joint work with Pieter Audenaert

HOL (Higher-order logic) systems [3] are a family of interactive theorem proving tools, widely used for creating formal specifications of systems, and for proving properties about them. They have been deployed in both industry and academia to support formal reasoning in many areas, including hardware and software verification. The underlying logic and basic facilities are completely general. In principle they can be used to support any project that can be defined in higher-order logic, an expressive logic originally developed as a foundation for mathematics. In [1] Audenaert extends this family with one more descendant: Formath. When formal methods are used to increase the reliability of software systems, the result directly depends on the correctness of the proof system that supports the application of the formal methods. It is shown that Formath is equiconsistent to Hol-4 and Hol-Light. Formath stands out from the latter, in using a deduction calculus based on sequents of the form $\Gamma_1 \vdash \Gamma_2$, where $\Gamma_1$ and $\Gamma_2$ represent finite sets of formulas, and in using a pert constant $C^{EXT}$ of type $(\alpha \to \beta) \to (\alpha \to \beta) \to \alpha$ in the formulation of the *Extensionality* axiom : $f(C^{EXT}fg) = g(C^{EXT}fg) \vdash f = g$. Note that $C^{EXT}fg$ represents a term $t$ such that $ft$ differs from $gt$ (if such a $t$ exists). Hence *Extensionality* reads as follows: two functions are equal if they coincide on their "difference". In [1] Formath is used for the specification and verification of Alfred programs (see [2]).

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ROSTISLAV HORČÍK

## Strong standard completeness of ΠMTL

Many-valued propositional logic ΠMTL is related to cancellative left-continuous t-norms (see [3]). It was obtained from monoidal t-norm based logic (MTL, see [1]) by adding two axioms ensuring cancellativity. MTL is a logic of left-continuous t-norms but it can be also viewed as an extension of $FL_{ew}$ (Full Lambek Calculus plus exchange and weakening) by adding $(\varphi \to \psi) \vee (\psi \to \varphi)$ as the axiom.
The corresponding algebras of truth values for ΠMTL are called ΠMTL-algebras. It was proved in [1] that ΠMTL is complete w.r.t. the class of linearly ordered ΠMTL-algebras. In [2], the authors studied whether ΠMTL is complete w.r.t. the class of ΠMTL-algebras in the real unit interval $[0, 1]$. However, they only succeeded in showing that ΠMTL is complete w.r.t. all ΠMTL-algebras whose universe is $\mathbb{Q} \cap [0, 1]$.
In this work we present a solution of this problem and show that ΠMTL is finitely strong standard complete, i.e., a finite theory $T$ over ΠMTL proves $\varphi$ iff $\varphi$ is true in all models over ΠMTL-algebras in $[0, 1]$.

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GÁBOR HORVÁTH

## Identities of finite groups

Let $\mathcal{A}$ be an algebra. Let us denote the difficulty of determining whether two terms are the same to all substitutions in $\mathcal{A}$ with $\mathrm{Term}-\mathrm{EQ}\,(\mathcal{A})$. Let us denote the same problem to polynomials (terms with constants) with $\mathrm{Pol}-\mathrm{EQ}\,(\mathcal{A})$. We also can check whether an equation can be satisfied in $\mathcal{A}$ or not, let us denote its difficulty with $\mathrm{Pol}-\mathrm{SAT}\,(\mathcal{A})$.
Burris – Lawrence and Szabó – Vértesi proved dihotomy for rings, the problems here are fully solved. For semigroups we almost know nothing, the few known results belong to McKenzie, Moore, Tesson, Thérien, Volkov, Szabó, Vértesi. In groups Burris – Lawrence proved that if $G$ is a nilpotent group, then the $\mathrm{Term}-\mathrm{EQ}\,(G)$ problem is in $P$, or if $G$ is non-solvable then $\mathrm{Term}-\mathrm{EQ}\,(G)$ is $coNP$-complete. Goldmann – Russel proved that solving system of equations is in $P$ for abelian groups and $coNP$-complete otherwise. We could prove the following:

**Theorem** (G. Horváth, Cs. Szabó)**.** *If $G \simeq A \rtimes B$ where $A$ is abelian and* $\mathrm{Term}-\mathrm{EQ}\,(B)$ *is in $P$ then* $\mathrm{Term}-\mathrm{EQ}\,(G)$ *is in $P$.*

This solves the problem e. g. for meta-abelian groups, $S_4$, $A_4$, $D_n$.
Let us denote with $\mathrm{Term}-\mathrm{EQ}^*$ and $\mathrm{Pol}-\mathrm{SAT}^*$ the former problems when we have the possibility to preprocess some new operations with the operations given by the algebra. Idziak – Szabó with TCT proved that in a congruence-modular variety $\mathrm{Pol}-\mathrm{SAT}^*\,(\mathcal{A})$ is in $P$ for nilpotent $\mathcal{A}$ algebras and $NP$-complete otherwise. This result does not say anything about terms. We proved that

**Theorem** (G. Horváth, Cs. Szabó)**.** *If $G$ is a nilpotent group then* $\mathrm{Pol}-\mathrm{SAT}^*\,(G)$ *is in $P$. If $G$ is not nilpotent then* $\mathrm{Term}-\mathrm{EQ}^*\,(G)$ *is $coNP$-complete.*

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HYKEL HOSNI

## Rationality as conformity

joint work with Jeff Paris

We suggest studying one aspect of individual rational choice where rationality is identified with the tendency to conform to the choices you expect other agents like you to make. Let $K$ be a finite set of options available to an agent. We characterise the behaviour of agents whose choice process consists firstly in picking those elements of $K$ that are believed, on the common knowledge of their rationality, to be the most common choice among the other agents, and then completing their choice by picking randomly from this set. Since we assume that agents cannot get together and agree on one particular choice, what we consider is in fact a form of a non-cooperative game.
A natural way of maximising choice agreement –when enforceable agreement is not an option– is to choose that subset $R(K)$ of $K$ that includes all and only the 'most distinguished' elements of $K$. We investigate two distinct characterisations of such a subset $R(K)$, firstly by means of principles of 'common-sense' in the sense of [Par94] and secondly in terms of minimising 'ambiguity'.

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BERNHARD IRRGANG

**Higher-Gap Morasses and Morass Constructions**

Higher-gap morasses were invented by Jensen in the early 1970's. He defined $(\kappa, \beta)$-morasses for regular $\kappa > \omega$ and $\beta < \kappa$. They are used to construct objects of size $\kappa^{+\beta}$ while the size of certain subobjects is kept below $\kappa$. Jensen's original application was the proof of the cardinal transfer theorem for higher gaps. In my talk, I will outline how the notion of morass can be generalized to $(\omega_1, \beta)$-morasses with $\beta \geqslant \omega_1$, and how these morasses may be used for constructions.

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Emil Jerabek

**Bounded arithmetic in 3-valued logic**

We present a bounded arithmetical theory over a variant of Kleene's 3-valued logic. The goal is to have a theory which can directly formalize basic counting arguments (we have approximate counting quantifiers in the language), moreover 3-valued logic is the natural framework for dealing with complexity classes of promise problems, such as promise-BPP. As it is the case with usual systems of bounded arithmetic, the theory permits only feasible reasoning — we prove that existential theorems of certain form are witnessable by probabilistic polynomial-time algorithms.

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Vladimir Kanovei

**Fully saturated extensions of standard universe**

joint work with Michael Reeken, Saharon Shelah

It seems that it has been taken for granted that there is no distinguished, definable, countably saturated nonstandard model of the reals. Of course $\mathbf{V} = \mathbf{L}$ implies the existence of such an extension (take the first one in the sense of the canonical well-ordering of $\mathbf{L}$), but the existence provably in **ZFC** was established quite recently in [1]. (Without Choice the existence of *any* elementary extension of the reals, containing an infinitely large integer, is not provable.) The existence of a definable fully saturated (that is $\kappa$-saturated for any cardinal $\kappa$) elementary extension of the whole set universe of **ZFC** is an even more challenging problem.

**Theorem 1.** *There exists, provably in* **ZFC**, *a definable fully saturated elementary extension of the whole set universe of* **ZFC**.

Such an extension can be viewed as an interpretation of bounded set theory **BST** in **ZFC**, such that the standard core of the interpretation coincides with the **ZFC** universe. (**BST** is an improved, foundations-friendly modification of internal set theory **IST**, in which every set is postulated to belong to a standard set.) Such an interpretation of **BST** was earlier obtained only on the base of Global Choice variants of **ZFC**. It is known that **IST** itself does not admit such an interpretation.

The proof of Theorem 1 consists of an Ord-long chain of consecutive iterated ultrapowers of the set universe of **ZFC**.

[1] V. Kanovei and S. Shelah, A definable nonstandard model of the reals, *JSL* 2004, 69, 1, 159–164.

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Asylkhan Khisamiev

## Quasiresolvable periodic Abelian groups

In [1], Yu. L. Eshov has introduced the definition of a quasiresolvable admissible set and has obtained the following results on such sets: if an admissible set $\mathbb{A}$ is quasiresolvable, then there exists a binary $\Sigma$-function which is universal for the class of unary $\Sigma$-function; if $\mathfrak{M}$ of a decidable model complete theory, then the admissible set $\mathbb{HF}(\mathfrak{M})$ is quasiresolvable. Definitions of a quasiresolvable model and of a 1-quasiresolvable model were introduced by the author in [2], where was described the quasiresolvable Abelian $p$-groups and the quasiresolvable Boolean algebras.
In this talk we introduce the notion of a $p$-quasiresolvable periodic Abelian group and describe the $p$-quasiresolvable periodic Abelian groups. We also obtain the condition of the quasiresolvable model in some class of a periodic Abelian groups.

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ANDRZEJ KISIELEWICZ

## A new solution to old paradoxes

DEST (Double Extension Set Theory) is a first-order theory with equality and two primitive binary predicates $\in$ and $\varepsilon$ (which are to be thought of as two different flavors of membership). Objects of our theory are called "sets"; but notice that some "irregular" sets may have two different extensions. A set $A$ is *regular* if $\forall x(x \in A \leftrightarrow x \,\varepsilon\, A)$. The system is based on one general

AXIOM SCHEMA OF COMPREHENSION. For all sets $x_1, \ldots, x_n$, if each of them is contained (in one sense or another) in a regular set with regular members, then

$$\exists A \forall x[(x \,\varepsilon\, A \leftrightarrow \varphi) \wedge (x \in A \leftrightarrow \overline{\varphi})],$$

where $\varphi$ is any formula making no use of the symbol $\varepsilon$, in which no variable other than $x, x_1, \ldots, x_n$ occurs free, and $\overline{\varphi}$ is its dual obtained from $\varphi$ by replacing each occurence of $\in$ by that of $\varepsilon$.

In addition we have

AXIOM OF MIXED EXTENSIONALITY. For all sets $A, B$,

$$\forall x(x \in A \leftrightarrow z \,\varepsilon\, B) \rightarrow A = B,$$

The system has survived various attempts to prove inconsistency, and recently, Randall M. Holmes has proven that the well-founded hereditarily regular sets in DEST model ZF. We discuss the way it avoids known antinomies and the problem of finding a relative consistency proof for DEST.

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Bernhard Koenig

## Partial Orderings with Dense Subtrees

We prove a couple of lemmas that basically give a complete characterization of when a poset contains a dense subtree and when not. As an application, it is remarked that it depends on cardinal arithmetic if posets like the "fast club" or "adding $\lambda$-many Cohen subsets of $\kappa$" have dense subtrees.

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NURLAN KOGABAEV

## Computable lists of $I$-algebras and complexity of some problems

In the present paper we study computable Boolean algebras with distinguished ideals. We briefly call them *$I$-algebras* and consider in the standard language for Boolean algebras extended by unary predicate symbols $I_1, \ldots, I_\lambda$, where $\lambda \in \omega$. A formal theory of $I$-algebras contains axioms of Boolean algebras and sentences stating that $I_1, \ldots, I_\lambda$ are ideals; $\lambda$ is fixed. A *computable list* of $I$-algebras is a uniformly computable sequence $(\mathcal{A}_n)_{n\in\omega}$ of $I$-algebras such that for any computable $I$-algebra $\mathcal{B}$, there is some $n$ with $\mathcal{B} \cong_c \mathcal{A}_n$. The structure of reducibility on computable lists of $I$-algebras (up to computable isomorphism) has the following properties:

**Theorem 1.** (1) *There is a principal computable list.*
(2) *There are countably many pairwise irreducible computable lists.*
(3) *There are no one-to-one, positive and negative computable lists.*

The existence of principal computable list of $I$-algebras $(\mathcal{A}_n)_{n\in\omega}$ allows us to associate any computable $I$-algebra with the index in this list (up to computable isomorphism) and to estimate the complexities of algorithmic problems on indices. The exact estimates for complexity of computable isomorphism problem, isomorphism problem and computable categoricity problem were obtained:

**Theorem 2.** (1) *The set* $\{\langle n, m\rangle \mid \mathcal{A}_n \cong_c \mathcal{A}_m\}$ *is* $\Sigma^0_3$*-complete.*
(2) *The set* $\{\langle n, m\rangle \mid \mathcal{A}_n \cong \mathcal{A}_m\}$ *is* $\Sigma^1_1$*-complete.*
(3) *The set* $\{n \mid \mathcal{A}$ *is computably categorical*$\}$ *is* $\Sigma^0_3$*-complete.*

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Juha Kontinen

**On definability of second order generalized quantifiers**

The notion of second order generalized quantifier was introduced in [1]. We consider definability of second order generalized quantifiers.
In the case of Lindström quantifiers, i.e., first order generalized quantifiers, definability of a quantifier $Q$ in a logic $\mathcal{L}$ just means that the class $Q$ is axiomatizable in $\mathcal{L}$. Since second order generalized quantifiers are classes of second order structures, there is no connection between quantifiers and classes of structures determined by sentences. Therefore, we introduce logics which are interpreted in structures consisting of a first order part and some second order predicates. Syntactically, these second order predicates act as first order quantifiers. We transform questions like "Can a logic $\mathcal{L}$ define $\mathcal{Q}$" to "Is $\mathcal{Q}$ the class of models of some $\psi \in \mathcal{L}'$", where $\mathcal{L}'$ is an extension of $\mathcal{L}$ by some second order predicates. It turns out that a defining sentence $\psi \in \mathcal{L}'$ can be converted into a uniform definition of $\mathcal{Q}$ in $\mathcal{L}$. So, definability of $\mathcal{Q}$ in $\mathcal{L}$ implies that $\mathcal{L}(\mathcal{Q}) \equiv \mathcal{L}$. Contrary to the first order case, $\mathcal{L}(\mathcal{Q}) \equiv \mathcal{L}$ does not imply definability.
Our main result is analogous to the Hierarchy Theorem in [2]:

**Theorem 1.** *Let $t$ be a second order type and $\mathbb{C}$ the collection of all second order generalized quantifiers of types lower than $t$. Then there is a quantifier $\mathcal{Q}$ of type $t$ which is not definable in the logic* $\mathrm{SO}(\mathbb{C})$.

Theorem 1 can be improved so that the undefinable quantifier $\mathcal{Q}$ is also monotone or recursive. Also, the version of Theorem 1 where $\mathcal{Q}$ in addition satisfies $\mathrm{FO}(\mathcal{Q}) \equiv \mathrm{FO}$ holds.

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Marcin Kozik

**A finite algebra with PSPACE-hard membership problem and at least exponential $\beta$ function**

joint work with R. McKenzie

We are going to modify a construction of R. McKenzie's A(T) to obtain a finite algebra which generates a variety with PSPACE-hard membership problem. We define the beta function of an algebra A, to be the function $\beta(n)$ such that its value is the minimal size of an identity required to witness the fact '**B** is not in HSP(**A**)' for any algebra **B** of size smaller then n. We present an algebra with the beta function growing at least exponentially.

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Jan Kraszewski

**On some properties of ideals on the Cantor space**

joint work with Marcin Kysiak

In 2001 Kraszewski in [1] defined a class of *productive* $\sigma$-ideals of subsets of the Cantor space $2^\omega$. Namely, a $\sigma$-ideal $\mathcal{J}$ is productive for every infinite $T \subseteq \omega$ and every set $A \subseteq 2^T$ if $A$ is in $\mathcal{J}$ then the cylinder $A \times 2^{\omega\setminus T}$ is in $\mathcal{J}$.

He observed that both $\sigma$-ideals of meagre sets $\mathcal{M}$ and of null sets $\mathcal{N}$ are in this class. Next, from every productive $\sigma$-ideal $\mathcal{J}$ one can produce a $\sigma$-ideal $\mathcal{J}_\kappa$ of subsets of the generalized Cantor space $2^\kappa$. In particular, starting from meagre sets and null sets in $2^\omega$ we obtain meagre sets and null sets in $2^\kappa$, respectively. This description gives us a powerful tool for investigating combinatorial properties of ideals on $2^\kappa$, which was done in [1].

However, some theorems needed an additional assumption that a $\sigma$-ideal of subsets of $2^\omega$ we started from had the Weak Fubini Property (which is a statement opposite to being productive, i.e a $\sigma$-ideal $\mathcal{J}$ has WFP if for every infinite $T \subseteq \omega$ and every $A \subseteq 2^T$ if the cylinder $A \times 2^{\omega\setminus T}$ is in $\mathcal{J}$ then its projection into $2^T$, that is $A$, is also in $\mathcal{J}$). The natural question posed in [1] was whether this extra assumption could be omitted.

The weaker version of this question is whether every productive $\sigma$-ideal of subsets of $2^\omega$ has the Weak Fubini Property. We answer it negatively. In the construction we make use of an interesting combinatorial lemma on existence of some special family of subsets of $\omega$.

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Mathis Kretz

**Deduction Chains for Logic of Common Knowledge**

joint work with Thomas Studer

Deduction chains represent a syntactic and in a certain sense constructive method for proving completeness of a formal system $\mathsf{S}$ with respect to the class of $\mathsf{S}$-structures. Given a formula $A$, the deduction chains of $A$ are built up by systematically decomposing $A$ into its subformulae. In the case where $A$ is a valid formula, the decomposition yields a (usually cut-free) proof of $A$ in the system $\mathsf{S}$. If $A$ is not valid, the decomposition produces a countermodel for $A$. The method of deduction chains was originally devised by Schütte and is mainly used in the context of first order logic and semiformal systems for various forms of arithmetic. Schütte [2] has also adapted the method to modal logic. In the current study, we extend this approach to a semiformal system for the Logic of Common Knowledge, as studied by Alberucci and Jäger [1]. The presence of fixed point constructs in this logic leads to potentially infinite-length deduction chains of a non-valid formula, in which case fairness of decomposition requires special attention. An adequate order of decomposition also plays an important role in the reconstruction of the proof of a valid formula from the set of its deduction chains.

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Krzysztof Krupiński

## A special thin type

In $SCF_{p,e}$ a thin but not very thin type of $U$-rank 1 is constructed.

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Oliver Kullmann

## The combinatorics of negation patterns

[2] introduced the *addressing problem* for graphs, considering labelling of nodes with code words over the alphabet $\{0, 1, *\}$ such that the distance between two nodes is reflected by the distance between their code words. The aim is to minimise the length of the code words, and their main result is a general lower bound on the length of the code words. It was noticed later, that the addressing problem can be generalised by considering a multigraph and the problem of *partitioning* the edge set into as few *bicliques* (complete bipartite graphs) as possible.
In [4, 5] these notions have been transferred to the area of boolean clause-sets, also opening new perspectives on the original problems. The starting point is that codes over $\{0, 1, *\}$ as well as biclique partitions of multigraphs are encodings of (boolean) clause-sets. [4, 5] is concerned with the basic notion of *hermitian rank* as introduced in [3] and its implications (especially for the theory of *autarkies*), while in [1] investigations have started how to exploit the new theory for satisfiability solving (SAT). I want to present an overview on these results and related open problems.

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Gábor Kun

## On the fundamental group of finite posets

Order-primal algebras were always in the center of algebraic research. They are good test-algebras for conjectures in universal algebra. The so-called Taylor-terms play the crucial role in their examination. The typical posets admitting no Taylor terms are the crowns. The nicest Taylor-terms on posets are the near-unanimity functions. By the works of Davey, McKenzie, Larose and Zádori we know that the existence of Gumm-terms, Jónsson-terms and a near-unanimity function are equivalent for order-primal algebras.

In our talk we give a combinatorial definition of the fundamental group of posets. We connect the algebraic and combinatorial properties of finite posets via a nice topological condition.

**Theorem 1.** *The fundamental group of every idempotent subalgebra of a finite poset is trivial iff no crown is retract of an idempotent subalgebra.*

**Corollary 2.** *If the finite poset $P$ admits Taylor-terms then the fundamental group of every connected idempotent subalgebra of $P$ is trivial.*

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Miloš S. Kurilić

**Ultrafilters in models of set theory**

While a forcing notion $\mathsf{P}$ can kill a mad family only on cardinals $\kappa < \mathrm{cc}(\mathsf{P})$ and produce independent subsets of $\kappa \leqslant \pi(\mathsf{P})$, $\mathsf{P}$ can kill uniform ultrafilters on each cardinal $\kappa \geqslant h_2(\mathrm{r.o.}(\mathsf{P}))$. For example, adding a real kills some ultrafilters on each cardinal and adding a Cohen or a random real kills all uniform ultrafilters on each cardinal less than the first measurable cardinal. Similarly, if a cardinal $\lambda$ obtains an independent subset and if $0^\sharp$ does not exist, then each uniform ultrafilter on each regular cardinal $\kappa \geqslant \lambda$ is killed. Some additional results concerning instability of ultrafilters in generic extensions will be given. Two typical results are:
1. Let $\lambda < \kappa$ be regular cardinals and let $\mathcal{B}$ be a base of some uniform ultrafilter on $\kappa$, which is $\lambda$-descendingly incomplete. If in a generic extension $V[G]$ there exists a cofinal increasing mapping $f : \lambda \to \kappa$, then in $V[G]$ $\mathcal{B}$ is not a base for an ultrafilter on $\kappa$.
2. Let $\kappa$ and $\lambda$ be infinite cardinals, let $\mathcal{U} \subset P(\lambda)$ and $\mathcal{V} \subset P(\kappa)$ be ultrafilters and let $V[G]$ be a generic extension. Then $\mathcal{U} \cdot \mathcal{V}$ is an ultrafilter base on $\lambda \times \kappa$ in $V[G]$ if and only if a) $\mathcal{U}$ and $\mathcal{V}$ are ultrafilter bases (on $\lambda$ and $\kappa$ respectively) in $V[G]$ and b) For each $\lambda$-sequence $\langle \mathcal{V}_\alpha : \alpha < \lambda \rangle \in V[G]$ of nonempty subsets of $\mathcal{V}$ there exists $U \in \mathcal{U}$ such that $\prod_{\alpha \in U} \{V \in \mathcal{V} : \exists V' \in \mathcal{V}_\alpha (V \subset V')\} \cap V \neq \emptyset$.

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Marcin Kysiak

## Nonmeasurable algebraic sums of sets of reals

A classical result by Sierpiński says that there exist sets $X, Y \subseteq \mathbb{R}$ of Lebesgue measure zero such that their algebraic sum, i.e. the set $X+Y = \{x+y : x \in X, y \in Y\}$, is not Lebesgue measurable (see [3]).
Some generalizations of this theorem can be found in the literature. Let $\mathcal{A}$ be a $\sigma$-algebra and $\mathcal{I} \subseteq \mathcal{A}$ a $\sigma$-ideal of subsets of $\mathbb{R}$. Under certain assumptions about $\mathcal{A}$ and $\mathcal{I}$ it can be shown that if $\mathcal{I}$ is not closed under algebraic sums then there exist $X, Y \in \mathcal{I}$ such that $X + Y \notin \mathcal{A}$. This was done by Kharazishvili in [2] assuming that the quotient algebra $\mathcal{A}/\mathcal{I}$ satisfies the countable chain condition and by Cichoń and Jasiński in [1] for every ideal $\mathcal{I}$ with a coanalytic base and the algebra $\mathcal{A} = Bor[\mathcal{I}]$.
In our talk we want to show that even a stronger assertion is true for a very wide class of algebras and ideals, including those considered by Cichoń and Jasiński as well as algebras and ideals naturally related to some popular forcing notions.
We will also discuss some additional related results concerning measure and category and the algebra of Borel sets.

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Gyesik Lee

## Kanamori-McAloon principle and the fast growing Ramsey functions

A. Kanamori and K. McAloon [1] introduced a principle a variant of Paris-Harrington theorem and showed that they are equivalent to each other. That is to say, two concepts "large" and "regressive" are connected, though the former concerns arithmetic functions and the latter sets of natural numbers. Both are responsible, respectively of course, for that two (true) principles are unprovable in $PA$.
Given an arithmetic function $F$ one defines $F$-large sets and $F$-regressive functions. Using a comparison with a fast growing hierarchy A. Weiermann [2] could characterize for what function class the generalized Paris-Harrington theorem remains $PA$-unprovable. Here it will be shown that such a characterization is also possible for the generalized Kanamori-McAloon principle concerning $F$-regressiveness.

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GIACOMO LENZI

**Axiomatizing bisimulation quantifiers**

joint work with Giovanna D'Agostino

Bisimulation quantifiers are used in the modal logic literature to prove results like uniform interpolation (a strong form of interpolation), see [1, 2, 3]. The existential bisimulation quantifier $\tilde{\exists} P \phi(P)$ holds in a graph $G$ if there is a graph $G'$ bisimilar to $G$ with respect to all atoms except $P$, and there exists a value of $P$ such that $\phi(P)$ is true in $G'$. Since the uniform interpolant of a formula can be expressed as a bisimulation-existential closure of the formula, it follows: if adding bisimulation quantifiers does not increase the expressive power of a logic, like in K or in modal $\mu$-calculus, then the logic has uniform interpolation. However, since the uniform interpolant is not expressed in the original language $L$, we cannot use an $L$-calculus to solve problems regarding uniform interpolants. One solution is to express the interpolant of a formula in $L$, by eliminating the quantifiers in an effective way; alternatively, one can try to axiomatize bisimulation quantifiers over $L$. On the other hand, there are logics where the addition of bisimulation quantifiers increases the expressive power of the logic: this is the case of Propositional Dynamic Logic, whose extension is equivalent to the $\mu$-calculus.

In this talk we present axiomatizations of bisimulation quantifiers over $K$, the $\mu$-calculus, and $PDL$. Moreover, for the class of disjunctive formulas of the first two logics we give an easy way to express the uniform interpolant in the original language.

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Stefano Leonesi

**On The Boolean Algebras of Definable Sets in Weakly O-minimal Theories**

joint work with Carlo Toffalori

We consider the sets definable in the countable models of a complete weakly o-minimal theory $T$ and we investigate under which conditions their Boolean algebras are isomorphic to each other; this is equivalent to say that each of these definable sets admits, if infinite, an infinite coinfinite definable subset.

We show that this is true if and only if $T$ has no infinite definable discrete (convex) subset.

Indeed, if this is false, then for every integer $n > 1$, or at least for every odd integer $n > 1$, there is some countable model $\mathcal{A}_n$ of $T$ including a discrete convex subset $D_n$ of order type $n \cdot (\omega^* + \omega)$ (so $n$ consecutive copies of $(\mathbb{Z}, \leqslant)$) which cannot be enlarged to the right or to the left by further copies of $(\mathbb{Z}, \leqslant)$.

Also we examine the same problems among arbitrary (complete) theories of mere linear orders, so excluding any additional structure and avoiding, of course, any direct reference to o-minimality or weak o-minimality. We prove some partial results in this setting.

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THIERRY LIBERT

**Topological solutions to the Fregean problem and their corresponding alternative set theories**

Given a topological space $U$, let $\mathscr{P}_*(U)$ equally stand for $\mathscr{P}_{cl}(U)$, the set of *closed* subsets of $U$, or for $\mathscr{P}_{op}(U)$, the set of *open* ones. Then, by a *weak topological solution* to the Fregean problem we mean a topological space $U$ together with a *bijection* $f : U \longrightarrow \mathscr{P}_*(U)$; and for a *strong* solution we require that $f$ be an *homeomorphism*, with respect to some natural topology on $\mathscr{P}_*(U)$. Cantor's theorem asserts that there is no *discrete* topological solution to the Fregean problem. The translation of this on the axiomatical side is, of course, Russell's paradox. But there are *non-discrete* topological solutions, weak or strong, and in this talk we shall relate the existence of these in connection with the axiomatic solutions to the set-theoretical paradoxes they offer (as in [3]). An example is given in [1] where the existence of strong topological solutions for positive *comprehension* with equality is established. We shall show that there are also natural strong topological solutions for positive *abstraction* without equality, which will yield a simpler proof of the consistency of extensionality with positive abstraction than the one given in [2] (see [4]).

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Sonia L'Innocente

**Minimalities and modules over rings related to Dedekind domains**

joint work with Vera Puninskaya and Carlo Toffalori

Several minimalities notions arise in the model theory of modules. Some of them (like strong, weak and quasi minimality) specify general concepts in the restricted framework of modules, others (such as pseudo-strong minimality) directly spring from this algebraic setting. A unifying notion (implying strong, pseudo-strong and quasi minimality) is a weak form of divisibility, requiring, for a module $M$ over a ring $R$, that, for every $r \in R$, either $Mr = 0$ or $Mr = M$.

We show that this property is just equivalent to quasi-minimality (up to elementary equivalence) over commutative rings $R$, but is properly weaker that quasi minimality in the noncommutative setting.
Then we characterize and compare all these notions over several classes of rings, including group rings $DG$ with $D$ a Dedekind domain and $G$ a finite group, pullbacks of Dedekind domains and Dedekind-like rings.

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Alexandre A. Lyaletsky (Jr.)

**On types of continuity defined by partially ordered sets and their interconnection with usual topological continuity**

Investigations are mostly devoted to new notions of convergence and continuity defined by partially ordered sets. Interconnection between these notions and their topological analogues (in particular, Scott's topologies constructed for semantics of $\lambda$-calculus) is given. Results obtained permit to consider the notions related to models of $\lambda$-calculus and to standard mathematical analysis from the unified point of view.

Let $\langle A, \leqslant \rangle$ be partially ordered set such that its every nonempty directed up (down) subset has the least upper bound (the greatest lower bound), and let $P$ be a sequence of its elements. Then for every increasing (decreasing) sequence over it, we define its *limit* as the least upper bound (the greatest lower bound) of its elements, and for every sequence $P$, we put its *limit* equal to such an element $a$ (if it exists) of $A$ that $a$ is the limit of every cofinal monotone subsequence of $P$ (if the set of all the cofinal monotone subsequents of $P$ is nonempty); otherwise limit of $P$ is undefined. An operation $f$ is called $\leqslant$-*continuous* iff $f$ preserves limits of all the sequences. Note that these notions generalize the usual notions of convergence and of continuity. The following results are obtained:

1. An operation $f$ over the partially ordered set $\langle A, \leqslant \rangle$ is $\leqslant$-continuous iff the following conditions are satisfied for every sequence $P$: (i) $f(limP)$ is defined $\Leftrightarrow$ $limf(P)$ is defined; (ii) the restriction of $f$ on every set, on which $f$ is monotone, is continuous w.r.t. a corresponding Scott topology.
2. It is not true that for every $\langle A, \leqslant \rangle$ belonging to some special class, there exists such a topology $T$ that the set of all $\leqslant$-continuous functions is equal to the set of all the $T$-continuous functions, and vice versa.
3. For a certain class of logics (which also contains the second-order classical logic), their exist reductions of the theories of $\leqslant$-convergence and of $\leqslant$-continuity to the theories of usual topological convergence and of usual topological continuity correspondingly.

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Maria Emilia Maietti

**Relative topology in constructive mathematics**

joint work with Silvio Valentini

Formal topology has been proposed by P. Martin-Löf and G. Sambin to deal with topological spaces within constructive mathematics (see [Sam03] for an update). By constructive mathematics we mean predicative intuitionistic mathematics whose foundation is Martin-Löf constructive set theory [NPS90].
The concept of formal topology is based on a binary cover relation and a unary positivity predicate. Together with the notion of continuous relation, formal topologies provide a predicative presentation of the category of open locales [JT84] and without the positivity predicate of that of locales and the two concepts are structural linked by a coreflection [MV04].
To deal predicatively with the *closed subsets* of a topological space, G. Sambin extended formal topology with a binary positivity predicate (see [Sam03]).
In this talk we furtherly extend formal topology with a binary positivity predicate by introducing *relative formal topology* to represent the topology induced on a closed subset and we connect the corresponding category with that of topological spaces and that of formal topologies via categorical adjunctions.

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Krzysztof Majcher

**Absolutely Ubiquitous Groups**

joint work with Aleksander Ivanov

Let $M$ be a countable structure over a finite language. Let $J(M)$ be the set of all isomorphic types of finite substructures of $M$. Then $M$ is *absolutely ubiquitous* if for any countable structure $N$ with $J(M) = J(N)$ we have $M \cong N$.

H.D.Macpherson has proved in [M] that absolutely ubiquitious structures are $\omega$-categorical, model complete and do not have the strict order property. He also shows that an absolutely ubiquitous group $G$ has a characteristic subgroup of finite index which is a direct product of elementary abelian groups of infinite rank.

We study absolutely ubiquitous groups in the language extended by automorphisms $L' = (\cdot, \alpha_1, ..., \alpha_n)$. We show that some stronger version the theorem of H.D.Macpherson holds in this situation: an absolutely ubiquitous $(G, \alpha_1, ..., \alpha_n)$ has a characteristic abelian subgroup $A < G$ of finite index with the following properties. Let $\gamma_1, ..., \gamma_m$ be the automorphisms of $A$ which are the conjugations induced by a transversal of $G/A$. Then $(A, \alpha_1, ..., \alpha_n, \gamma_1, ..., \gamma_m)$ is absolutely ubiquitous. As a module over $R = \mathbf{Z}[\alpha_1, ..., \alpha_n, \gamma_1, ..., \gamma_m]$ the group $A$ is a direct sum of finitely many $R$-modules of the form $B^\omega$, where $B$ is irreducible and does not have non-trivial submodules (this follows from absolute ubiquity of $A$).

We apply these results to some characterisation of absolutely ubiquitous groups in terms of their submodules of finite index.

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Hamid Reza Maleki

**An Algorithm for Solving Multiobjective Linear Programming**

joint work with Maryam Zangiabadi, Mohammad Reza Safi, and Mashaalah Mashinchi

Generally, An engineering design problem has multiobjective functions. Some of these problems can be formulated as a multiobjective linear programming model. In this paper we use a fuzzy approach for solving multiobjective linear programming problems. Then we propose an algorithm for solving it based on our approach. Also we give an example for illustrating performance of the proposed approach.

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Gian Arturo Marco

## Invariant model theory and automorphism group actions

This research answers model theoretic questions from the point of view of the invariant descriptive set theory of closed subgroups of $\omega!$ (1): this approach justifies the title of the paper. We give evidence for the existence of a deep connection between two apparently far apart problems: the coordinatization conjecture (2) and the back and forth asymmetry problem (3). The former is a problem in extendent model theoretic definability, the latter concerns the characterisation of a proper subclass of back and forth processes induced by the action of automorphism groups on definable pointsets. The main results of this paper are the following:

i) we prove the coordinatization conjecture for closed left invariant automorphism groups and
ii) the asymmetry property for Jordan automorphism groups.

Moreover we show that Jordan automorphism groups form an extension of closed left invariant groups in their relative topology.

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Petar Markovic

**Star-linear varieties of groupoids**

joint work with P. Djapic, J. Jezek, R. McKenzie and D. Stanowski

We define an equational theory $E$ of groupoids to be $*$-*linear* if every term is equal to precisely one linear term (a term in which each variable occurs at most once). We show that there are precisely six such equational theories. Moreover, if we only assume that any term is equal to a linear term, not necessarily unique, (we call these equational theories $*$-*quasilinear*), we obtain a full description of such idempotent equational theories, while in the non-idempotent case we show that there are infinitely many $*$-quasilinear equational theories, not all of which are locally finite.

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Ralph McKenzie

**Finite decidability and generative complexity in equational classes**

The generative complexity of a class $\mathcal{K}$ is the function $G_{\mathcal{K}}$ defined for positive integers $n$ with $G_{\mathcal{K}}(n)$ equal to the number of non-isomorphic structures in $\mathcal{K}$ generated by at most $n$ elements. With P. Idziak and M. Valeriote, we proved that for a locally finite variety (i.e., equational class) $\mathcal{K}$ of algebras, $G_{\mathcal{K}}(n)$ is bounded by a polynomial function of $n$ if and only if $\mathcal{K}$ decomposes into a varietal product of an affine variety over a ring of finite representation type, and a sequence of strongly Abelian varieties equivalent to matrix powers of varieties of $H$-sets, with constants, for various finite groups $H$. A class $\mathcal{K}$ of structures is said to be finitely decidable if its class of finite members has decidable first order theory. Some arguments from the proof of the mentioned characterization of locally finite varieties with polynomially many models have been adapted by the author to prove several conjectured properties that the congruences relations must satisfy in any finite algebra belonging to a finitely decidable variety. A hoped for structural characterization of all finitely decidable, finitely generated varieties is still being sought. The talk will survey the mentioned results and provide some details.

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ROBERT K. MEYER

## Boole Bites Back

Meyer and Routley conservatively extended the relevant logic $R$ to accommodate Boolean negation $-$ in the system here called $CR$. (See [1].) Speaking on behalf of their "Relevantist", a simple majority of its principal authors expanded an earlier paper to attack Boolean negation in [2]. Like atomic gadgets and assorted rigamarole, including mouthwash, the relevantist thinks Boolean negation should not have been invented. Let those who will speak up for the atom; as for mouthwash, it all depends upon whom one is biting tonight. And tonight, in this talk, Boole bites back. [2] argues, for example, that the original equipment DeMorgan $\sim$ is an INTERNAL negation, as opposed to the merely EXTERNAL Boolean $-$. It is true, I concede, that Boolean $-$ may be defined via the semantically important Routley $*$ by $-A = \sim * A$ ($= *{\sim}A$, if you'd rather). The problem is that this argument can be cranked the other way, taking Boolean $-$ as one's preferred "internal" negation and then defining its "external" DeMorgan cousin by ${\sim}A = - * A$ (equivalently, $= * - A$). What this paper will argue is that there is ample relevant room for BOTH negations. Absence of Evidence is not to be confused with Evidence of Absence.

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H.G.R. Millington

## Mathematical Foundations in a Lambda Calculus Extension of Łukasiewicz Logic

We give a brief description of a formal language which offers a resolution of the foundational paradoxes of mathematics, yields a finitistic proof of the consistency of axiomatic set theory, and enables the development of the same up to and including a theory of categories. The language is based on a lambda calculus extension of Łukasiewicz infinite-valued propositional logic.

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Ryszard Mirek

## Local Finiteness and Substructural Algebras

According to Malcev [3] a universal algebra $\mathcal{A} = (A, \{f_i\}_{i \in I})$ is said to be *locally finite* if every finitely generated subalgebra of $\mathcal{A}$ is finite. A class $\mathcal{K}$ of (the same type) universal algebras is called *locally finite* if every algebra from $\mathcal{K}$ is locally finite. If a class $\mathcal{K}$ is locally finite, then every finitely generated algebra from $\mathcal{K}$ is finite, but the converse of this statement is not generally true. G. Bezhanishvili in [1] calls such classes *locally finite in the weak sense*. Moreover, he extends Malcev's criterion to varieties of infinite signature and simultaneously restricts it to the class $V_{FGSI}$, finitely generated subdirectly irreducible $V$-algebras. A variety is locally finite iff the class $V_{FGSI}$ is regularly locally finite. If $V$ is of finite signature it is enough if the class $V_{FGSI}$ is uniformly locally finite.

My aim is to survey some substructural algebras using Bezhanishvili's criterion as well. For instance it is known that every *finite residuated lattice* is $k$-potent for some positive integer $k$. It means that it satisfies $E_k$ ($E_k : x^{k+1} = x^k$ for any $x$.) [4]. Moreover, if a subdirectly irreducible residuated lattice satisfies $E_k$ for some $k$, then it has unique coatom. Every finite subdirectly irreducible residuated lattice has a single coatom. One can ask whether every locally finite residuated lattice has a coatom. Generally the answer is negative. The counterexample is free generated Boolean algebra with countable number of generators. As concerns (sub)varieties, any free algebra of the variety of algebras determined by the purely implicational fragment of all theses of the intuitionistic propositional calculus is locally finite, but it is not true in BCK algebras [2].

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Erich Monteleone

## Thoughts in the age of computer science

We try to establish a relation among philosophical doctrines and computer science concepts that express deep scientific truths. We take an example introducing solipsism and algorithmic complexity theory.

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Georg Moser

## A Note on Cichon's Principle

We study Cichon's Principle (CP) [3] in the context of Knuth-Bendix orders (KBOs). Let $\prec_{\text{KBO}}$ denote an instance of KBOs and let $\varphi$ denote the fix-point free Veblen-function. For any term $t$ in the ground term algebra $\mathcal{T}(\Sigma)$ we define an interpretation $o\colon \mathcal{T}(\Sigma) \to \varphi(\omega, 0)$ such that $o$ allows an approximation of $\prec_{\text{KBO}}$ by the Buchholz-relation $<^B_k$ [2, 1]; $k$ depends on $R$ only. Employing the point-wise function

$$\mathrm{G}_k(\alpha) := \max\{n < \omega\colon \exists(\alpha_0, \ldots, \alpha_n)[\alpha_n <^B_k \cdots <^B_k \alpha_0 = \alpha]\}$$

an (essential) optimal worst-case complexity analysis of $R$ is possible. The advantage being that we are spared the (lengthy) calculations of [4]. Furthermore—in the restricted case of a ground rewrite system—we obtain stronger bounds. (Similar results hold in the general case.) Now let $N$ denote Cichon's maximum norm, cf. [3]. Set

$$\alpha <^C_k \beta :\Leftrightarrow (\alpha < \beta \wedge N(\alpha) \leqslant N(\beta) + k).$$

We define an interpretation $\pi\colon \mathcal{T}(\Sigma) \to \omega^\omega$ such that $\prec_{\text{KBO}}$ can be approximated by $<^C_k$ and the point-wise function

$$\mathrm{A}_k(\alpha) := \max\{n < \omega\colon \exists(\alpha_0, \ldots, \alpha_n)[\alpha_n <^C_k \cdots <^C_k \alpha_0 = \alpha]\}$$

yields a worst-case complexity analysis of $R$. The latter result is in accordance (with some formulations of) CP as the maximal order-type of any KBO is $\omega^\omega$. However $\mathrm{A}_k(\omega^\omega)$ majorizes the Ackermann function. This follows from the known lower-bounds of the derivation length function for $R$, cf. [4].

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Marcin Mostowski

**Arithmetic of coprimality in finite models**

joint work with Konrad Zdanowski

Let $FM(\mathcal{A})$, for $\mathcal{A} = (\omega, \ldots)$ be a family of finite, initial segments of $\mathcal{A}$. A relation $R$ is $FM$–representable in $FM(\mathcal{A})$ if there is a formula $\varphi$ which correctly describes each finite fragment of $R$ in almost all models from $FM(\mathcal{A})$ (see [M1, M2] and the following papers [KZ, MZ, MW]). We investigate $FM(\mathcal{A})$ for $\mathcal{A}$ being a standard model for arithmetic of coprimality, $(\omega, \perp)$.

A relation $R$ is coprimality invariant if being coprime with the same numbers is a congruence for $R$. We show that $R \subseteq \omega^n$ is $FM$–representable in $FM(\omega, \perp)$ if and only if $R$ is coprimality invariant and $R$ is $FM$–representable in $FM(\omega, +, \times)$. As a consequence we obtain that $FM(\omega, +, \times)$ is interpretable in $FM(\omega, \perp)$ and that $Th(FM(\omega, \perp))$ is $\Pi_1$–complete (this improves the theorem for divisibility relation from [MW]). Moreover, in the above results we do not use the equality predicate in $FM(\omega, \perp)$.

Our method gives also that $(\omega, +, \times)$ is interpretable in $(\omega, \perp, \leqslant_{P_3})$, where $\leqslant_{P_3}$ is standard ordering restricted to the set of products of three different primes (compare with [BR]).

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Nebojša Mudrinski

## Some Results on Endomorphism Monoids of Countable Homogeneous Graphs

joint work with I. Dolinka

A first-order structure $M$ is called *homogeneous* (or *ultrahomogeneous*) if every isomorphism $\varphi_0 : F_1 \to F_2$ between its finitely generated substructures $F_1, F_2$ can be lifted to an automorphism $\varphi$ of $M$, so that $\varphi|_{F_1} = \varphi_0$. In relational structures (e.g. in graphs), since every subset of the domain of $M$ carries an induced substructure of $M$, in the above definition one may replace the words 'finitely generated' by 'finite'.

Countable homogeneous graphs were completely classified in 1980 by Lachlan and Woodrow. These are: $mK_n$ ($m$ disjoint copies of complete graphs of size $n$), where at least one of $m, n$ is infinite, the complements of these, the *Henson graphs* $H_n$, $n \geqslant 3$ (the homogeneous countable graphs universal for the class of all finite graphs omitting a clique of size $n$), the complements of Henson graphs, and the countable random graph $R$. Also, there is a number of other classes in which the homogeneous structures have been determined (including, for example, finite graphs, countable tournaments, digraphs, posets and finite groups).

In this talk, we investigate the algebraic properties of endomorphisms of the graphs just described, and give an account on the following recent results:

- The automorphism group of every countable homogeneous graph contains a copy of each countable group. From that fact we deduce undecidability of the (universal) theory of such a group.
- In passing, we show that the Henson graphs $H_n$ are retract rigid.
- We present some properties and charaterizations of retracts of complements of Henson graphs. Also, there are several results on the partially ordered set of idempotent endomorphisms of these graphs.

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Pavel Naumov

## On Modal Logic of Deductive Closure

The classical propositional logic is known to be sound and complete with respect to the set semantics under which propositional connectives conjunction, disjunction, and negations are interpreted as operations intersection, union, and complement on subsets of a universe $U$. In particular, this is true when the universe is a set of propositions in some language. Here we consider an additional connective $\Box$ which, under the above interpretation, corresponds to the deductive closure operator on the universe of propositions $U$ with respect to a theory $T$. By logic of deductive closure $\mathcal{D}$ of theory $T$ we mean the set of all propositional modal formulas which, for any interpretation of propositional variables, are equal to the entire universe $U$.

**Theorem 1.** *The modal logic of deductive closure of classical propositional logic is an extension of the classical propositional logic by the following modal axioms and inference rules*

- *reflexivity:* $\phi \to \Box\phi$,
- *transitivity:* $\Box(\phi \vee \Box\phi) \to \Box\phi$,
- *monotonicity:* $\dfrac{\phi \to \psi}{\Box\phi \to \Box\psi}$.

**Theorem 2.** *The modal logic of deductive closure of classical propositional logic is decidable.*

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SARA NEGRI

## A uniform cut elimination theorem for contraction-free systems of modal logic

A method is presented by which cut elimination can be established in one theorem for systems encompassing most normal modal logics. Contraction is an admissible rule in all the systems considered.
By a slight variation of the method, syntactic cut elimination obtains also for a contraction-free system for Gödel-Löb provability logic.

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HASSAN MISHMAST NEHI

**A Canonical Representation for the Solution of Fuzzy Linear System and Fuzzy Linear Programming**

joint work with Hamid Reza Maleki and Mashaalah Mashinchi

In this paper first, we find a canonical symmetrical trapezoidal (triangular) for the solution of the fuzzy linear system $A\tilde{\mathbf{x}} = \tilde{\mathbf{b}}$, where the elements in $A$ and $\tilde{\mathbf{b}}$ are crisp and arbitrary fuzzy numbers, respectively. Then, a model for fuzzy linear programming problem with fuzzy variables (FLPFV), in which, the right hand side of constraints are arbitrary numbers, and coefficients of the objective function and constraint matrix are regarded as crisp numbers, is discussed. A numerical procedure for calculating a canonical symmetrical trapezoidal representation for the solution of fuzzy linear system and the optimal solution of FLPFV, (if there exist) is proposed. Several examples illustrate these ideas.

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## On lattices of varieties of algebras

Let $V$ be variety of algebras. A set $Lv(V)$ of all subvarieties of $V$ forms co-algebraic lattice under inclusion. A lattice $L$ is called a lattice of varieties if there exist variety $V$ such that $L$ is isomorphic to $Lv(V)$. It is well-known Birkhoff-Malcev Problem: Characterize those lattices that can be represented as lattices of varieties. In the present work we construct a class of algebras, so-called $Lv$-monoid algebras, which signature consists one monoid operation with unit as constant and two unary operations and show that any lattice $L$ is a lattice of varieties if and only if $L$ is dually isomorphic to lattice of congruences of some $Lv$-monoid algebra.

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## Model Theory and Philosophy of Language

The development of formal semantics in the last decades, shows two forms of departure from what Jaroslav Peregrin has called the Carnapian paradigm (1), whose fundamental tenet was that (meaningful) language is to be understood as a (formal) system of names of things. This general conception of language led naturally, with the aid of model theory, to the study of the relationships between a linguistic and an extra-linguistic realm. As philosophers became more aware of the inadecuacy of the original Carnapian paradigm in providing a formal semantics for natural languages, several attempts were made, within the framewok of model theory, in order to account for features (such as indexicality and synonymy) characteristic of natural languages. Among such attempts, I distinguish between those that uncritically presuppose a radical distinction between the linguistic and the non-linguistic realm, and those that do not. In this paper I present Peregrin's inferentialist view of semantics as an example of this latter form of departure from the Carnapian paradigm. According to Peregrin, model theory can be considered a theory of semantics, not because it can imitate the "real" denotandum/denotatum relationship, but because model theory can serve to explicate consequence (2). I then explain why Peregrin's argument for this claim seems inconclusive, and conclude by suggesting one way in which his main underlying thesis may be substantiated.

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Giovanni Panti

**Substitutions as dynamical systems**

Up to equivalence, a substitution in propositional logic is an endomorphism of a free algebra. On the dual space, this results in a continuous function, and whenever the space carries a natural measure one may ask about the stochastic properties of the action. In classical logic there is a strong dichotomy between the finite and the infinite-variable cases: the first case is trivial, while the second is definitely not so.
In many-valued logic this dichotomy disappears: already in the finite-variable case many interesting phenomena occur. We are mainly concerned with the Łukasiewicz infinite-valued logic. We classify the 1-variable substitutions in Łukasiewicz logic which are ergodic with respect to Lebesgue measure, and we prove that a substitution has dense orbits for Lebesgue all points iff it is ergodic. This property fails for other many-valued logics, such as Hajek product logic.
We prove that in Łukasiewicz logic two variables are sufficient to obtain extremely strong stochastic properties. As a matter of fact, we construct a family of substitutions in 2-variable Łukasiewicz logic that have the Bernoulli property, i.e., are measure-theoretically isomorphic to a biinfinite sequence of independent identically distributed random variables.

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FRANCO PARLAMENTO

**Truth in V for $\exists^*\forall\forall$-sentences is decidable**

joint work with Dorella Bellè

Let $V$ be the cumulative set theoretic hierarchy, generated from the empty set by taking powers at successor stages and unions at limit stages and, following [2], let the primitive language of set theory be the first order language which contains binary symbols for equality and membership only. Despite the existence of $\forall\forall$-formulae in the primitive language, with two free variables which are satisfiable in $V$ but not by finite sets ([3]), and therefore of $\exists\exists\forall\forall$ sentences of the same language, which are undecidable in ZFC without the infinity axiom, both truth in $V$ and in $V_\omega$, for $\exists^*\forall\forall$-sentences of the primitive language, are decidable ([1]).

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## A useful reformulation of first-order classical logic

Let us assume that constant symbols occupy quantifier positions instead of argument postions as they do in the classical first-order language (the latter positions being reserved for variables). Specifically, if $a$ is a constant symbol, $x$ is a variable, $A$ is a formula then the expression $axA$ is also a formula. Such a syntactic regimentation of subject-predicate formulas may seem awkward and unnecessary complicated. However, the resulting formal language enables us to formulate the following deductive system, call it $F$, in which constants can represent not only proper names:

**M1** Classical truth-functional tautologies.
**M2** $\forall x(A \supset B) \supset (axA \supset axB)$
**M3** $A \supset \forall\, xA$, provided $x$ is not free in $A$.
**M4** $\forall xA \supset A(y|x)$
**M5** $axA \supset ayA(y|x)$, provided y is not free in $A$.
**M6** $x{=}x$
**M7** $x{=}y \supset (A \supset A(y\|x))$
**MP** if $\vdash_F A \supset B$ and $\vdash_F A$ then $\vdash_F B$
**MG** if $\vdash_F A$ then $\vdash_F axA$ and $\vdash_F \neg ax \neg A$

The semantics for $F$ coincides with the classical set-theoretic semantics with respect to a non-empty domain of interpretation $D$, interpretations of predicates, truth conditions for atomic formulas, negation and implication. Any constant $a$ is interpreted as a generalised quantifier, $I(a) \subset \mathrm{Exp}(D)$. A subject-predicate formula $axA$ is true iff there exists $X \in I(a)$ such that $X \subset [d\colon\ A$ is true if $d$ is assigned to $x]$. The logic $F$ can accommodate a vast class of noun phrases including proper names, definite and indefinite descriptions (even such like *the round square*), increasing monotone quantifiers (eg. the Rescher quantifier), plurals. The logic $F$ can be further extended to a higher-order logic based on the same alphabet as $F$.

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## On pressing down game and Banach–Mazur game

joint work with Saharon Shelah

We consider a proper ideal $I$ on an uncountable regular cardinal $\kappa$. Recall the Banach–Mazur game on $I$, studied in [2], denoted here by $BM_\gamma(S)$. It has two players, I and II, who alternately choose a descending $\gamma$–sequence of $I$–positive sets. I starts this game with a subset of the $I$–positive set $S$, and II wins if she can play all $\gamma$ rounds. The ideal $I$ is called *precipitous* iff I does not win $BM_{\omega+1}(\kappa)$.
Related to this game is the pressing down game, denoted $PDG_\gamma(S)$, defined in [1]. During each round $i < \gamma$, I first chooses a regressive function $f_i$ on $\kappa$ and II responds with a subset $S_i$ of $\bigcup_{j<i} S_j$ (or $S$ if $i = 0$) such that it is unbounded in $\kappa$ and $f_i$ is constant on $S_i$. II wins if she can play all $\gamma$ rounds.
It is clear that if II wins $BM_\gamma(S)$ then II wins $PDG_\gamma(S)$. We show that it is consistent that the reverse implication does not hold. This we obtain when $\kappa$ is a measurable cardinal, forcing a winning strategy for $I$ in $BM_\gamma(S)$ and using the normality of the ultrafilter given by $\kappa$. We also discuss the situation where the measurable $\kappa$ is collapsed to $\aleph_2$ and then do the forcing.

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Marcin Petrykowski

**Coverings of groups**

joint work with Ludomir Newelski

Assume $G$ is an $\aleph_0$-saturated group covered by countably many 0-type-definable sets $X_n$, $n < \omega$. Earlier we have shown that $G$ is generated by finitely many of the sets $X_n$ in 2 steps (when $G$ is stable) or in 3 steps (in general). Now we improve this as follows. We call a definable set $A \subseteq G$ weak generic if there exist a definable generic set $B$ and a definable non-generic set $C$ such that $A = B \setminus C$. A complete type $p$ is weak generic if every formula from $p$ defines a weak generic subset of $G$.

**Theorem 1.** *If $X_n \cap$(weak generic types) has non-empty relative interior in (weak generic types) then $G = A \cdot X_n \cdot X_n^{-1}$ for some finite $A \subseteq G$.*

We have also found a new way to prove that in the stable case $G = X_{\leqslant n} \cdot X_{\leqslant n}^{-1}$ for some $n < \omega$. The idea involves some measure on the algebra of definable subsets of $G$. This leads to a similar result on amenable groups (that is groups $G$ with a finitely additive left-invariant measure on $\mathcal{P}(G)$).

**Theorem 2.** *If $G$ is stable or amenable then $G = X_{\leqslant n} \cdot X_{\leqslant n}^{-1}$ for some $n < \omega$.*

These theorems have counterparts outside model theory. Namely, let $X$ be a compact space covered by closed sets $X_n$, $n < \omega$, and let $f\colon G \to X$ be any function.

**Theorem 3.** *There exist $n < \omega$ and some finite $A \subseteq G$ such that for every open $U \supseteq X_n$*

$$G = A \cdot f^{-1}[U] \cdot (f^{-1}[U])^{-1}.$$

**Theorem 4.** *If $G$ is amenable then there exists $n < \omega$ such that for every open $U \supseteq X_{\leqslant n}$*

$$G = f^{-1}[U] \cdot (f^{-1}[U])^{-1}.$$

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DAVID PIERCE

**Fields of any characteristic with operators**

For the theories of fields of specified characteristic with a derivation $\delta$, the respective model-companions $\mathrm{DCF}_0$ and $\mathrm{DCF}_p$ are known. Their intersection is a model-complete theory DCF, whose models are just those differential fields $(K, \delta)$ such that (0) $K$ is separably closed; (1) $K^p$ is the constant-field of $\delta$, if $\operatorname{char} K = p > 0$; (2) for every variety $V$ over $K$, if $\phi$ and $\psi$ are rational maps from $V$ into a (finite-dimensional) affine space, $\phi$ being dominant and separable, then $\delta(\phi P) = \psi P$ for some $K$-rational point $P$ of $V$:

$$\begin{tikzcd} V \arrow[d, "\phi"'] \arrow[dr, "\psi"] & \\ \mathbb{A}^n \arrow[r, dotted] & \mathbb{A}^n \end{tikzcd}$$

For the theory of fields with an endomorphism $\sigma$, the model-companion ACFA is also known. It can be given axioms like those of DCF. Yet there is no model-companion of the theory of fields equipped with both $\delta$ and $\sigma$, except in characteristic zero. Indeed, if $(K, \delta, \sigma)$ is existentially closed, and $K$ has positive characteristic $p$, then $K/K^\sigma$ is a proper but purely inseparable extension, and $K^p = \{x \in K : \bigwedge_{n\in\omega} \delta(x^{\sigma^n}) = 0\}$; these are not first-order conditions.

The above formulation of DCF generalizes to $\mathrm{DCF}^m$, the model-companion of the theory of fields with $m$ commuting derivations. The fundamental idea is that such derivations of $K$ span a finite-dimensional subspace and *Lie sub-ring* $E$ of $\mathrm{Der}(K)$. This determines a derivation $\mathrm{d} : K \to E^*$ where $\mathrm{d}\, x$ is $\delta \mapsto \delta x$, and an additive map $\mathrm{d} : E^* \to (E \wedge E)^*$ where $\mathrm{d}\, \theta$ is $\delta_0 \wedge \delta_1 \mapsto \delta_0(\delta_1\theta) - \delta_1(\delta_0\theta) - [\delta_0, \delta_1]\theta$. The latter d is also a sort of derivation, and $\mathrm{d}^2 = 0$. If $L$ is a field extending $K$, then any embedding of $E$ in $\mathrm{Der}(L)$ corresponds to an extension of the maps d. This allows (2) above to be formulated for the more general case.

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Michael Pinsker

**Set theory in infinite clone theory**

Let $X$ be a set. A *clone* is a set of finitary functions on $X$ which is closed under composition and which contains all projections. Equivalently, a clone can be defined to be the set of term operations of a universal algebra on $X$. The set of all clones on $X$ forms a complete algebraic lattice with respect to inclusion.
Whereas the investigation of the clone lattice on finite $X$ requires methods from algebra, it involves mainly tools from set theory if $X$ is infinite. Examples of applications of set theory to clones are the use of descriptive set theory in [1] and the forcing-inspired construction in [2].
In [4], we give for all $X$ of infinite regular cardinality we give an explicit list of all dual atoms $\mathscr{C}$ of the clone lattice whose unary part $\mathscr{C}^{(1)}$ is non-trivial and contains all permutations of $X$. It turns out that the number of such clones is relatively small and a monotone function of the cardinality of the base set $X$. This is interesting in the light of another result [3] stating that the number of dual atoms which contain all unary functions varies heavily with the partition properties of $X$ and is highly non-monotone.

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## Uniform refinement properties in distributive semilattices

There is a well-known problem in the lattice theory called the Congruence Lattice Problem (CLP): Is every distributive algebraic lattice isomorphic to congruence lattice of some lattice?
There are some partial positive and negative results. The negative results are connected with various uniform refinement properties, first discovered by F. Wehrung about ten years ago. These properties are based on infinite combinatorics, especially the Kuratowski principle about free sets.
So far, none of these properties could solve the CLP in full. Now we introduce a new uniform refinement property, weaker then the previous ones, and investigate its potential for soving the CLP and related problems.

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S. Yu. Podzorov

## Rogers semilattices of arithmetical numberings

A *numbering* of a set $S$ is an arbitrary surjective mapping from $\mathbb{N}$ onto $S$. For two numberings $\nu$ and $\mu$ we say that $\nu$ is *reducible* to $\mu$ if there exists a computable function $f$ such that $\nu = \mu f$. All numberings of $S$ with reducibility relation form a preordered structure; it's quotient modulo the induced equivalense forms an upper semilattice $\mathcal{R}(S)$.

Let $S$ be the family of subsets of $\mathbb{N}$. A numbering $\nu$ of $S$ is called $\Sigma^0_n$-*computable* if the set $\{\langle x, y\rangle : x \in \nu y\}$ is in the class $\Sigma^0_n$ of arithmetical hierarchy. We say that numbering is *arithmetical* if it is $\Sigma^0_n$-computable for some $n$. Subsemilattice of $\mathcal{R}(S)$ defined by all $\Sigma^0_n$-computable numberings is called *Rogers semilattice* and denoted $\mathcal{R}^0_n(S)$.

We investigate algebraic properties of Rogers semilattices for several years. Different kind of results was obtained. We mention three of them below.

(1) It is proved that for any $S$ and any $n \geqslant 2$ an arbitrary Lachlan semilattice (i.e. principal ideal of c.e. $m$-degrees) could be embedded in $\mathcal{R}^0_n(S)$ if and only if $\mathcal{R}^0_n(S)$ is nonempty and has more than one element.

(2) Some sufficient conditions for minimal coverings in $\mathcal{R}^0_n(S)$ for $n \geqslant 2$ were found. As a corollary we obtained that if $S$ is finite or has minimal element under inclusion than any nongreatest element of $\mathcal{R}^0_n(S)$ has a minimal covering.

(3) We say that the element of $\mathcal{R}^0_n(S)$ has a *limitness property* if it is not a minimal covering of any other element. It is suspected that the greatest element of $\mathcal{R}^0_n(S)$ (if it exists) always has a limitness property. We proved that this statement holds if $S$ is finite, contains finite set or has the least element under inclusion.

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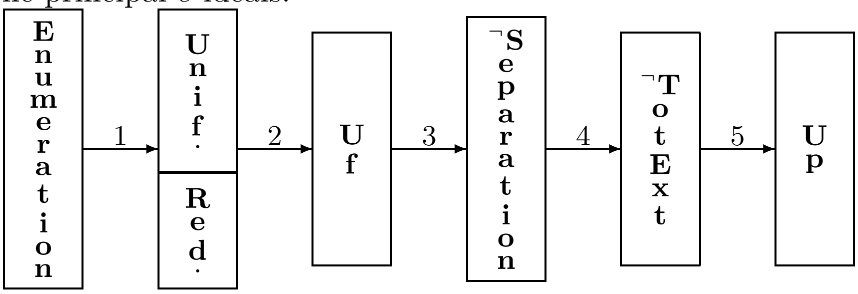
VADIM PUZARENKO

**On computability in structures**

joint work with I. Kalimullin

We establish connections between computability in enumeration degrees ideals and computability in admissible sets. More exactly, considered below problems concern computable properties of collection of all the c.e. (or $\Sigma$) subsets of $\omega \subseteq \mathbb{A}$ (set consisting of all the natural ordinals) where $\mathbb{A}$ is an arbitrary admissible structure. Given an admissible set $\mathbb{A}$, we denote collection of e-degrees of all the $\Sigma$ subsets of $\omega$ on $\mathbb{A}$ as $I_e(\mathbb{A})$. Notice that $I_e(\mathbb{A})$ is an $e$-ideal. Most of the properties on ideals and admissible sets have natural sense. We consider the following properties from descriptive set theory: Enumeration, Reduction, Uniformization, Separation, Total Extension and Existence of an Universal Function and of an Universal Predicate.

**Theorem 1** The following relations hold for the properties on collection of all the principal $e$-ideals:




Any dividing $e$–degree for $1, 5$ is non-arithmetical; there is a dividing $\Sigma^0_2$-$e$-degree for $2, 3, 4$. Furthermore, there is a quasiminimal dividing $e$-degree for $2, 3$; there is no quasiminimal dividing $e$-degree for $1, 4, 5$.

**Theorem 2** Given any enumeration degree $\mathbf{a}$, there exists a model $\mathfrak{M}_\mathbf{a}$ in a finite language with $I_e(\mathbb{HF}(\mathfrak{M}_\mathbf{a})) = \widehat{\mathbf{a}}$ such that $\varphi(\mathbb{HF}(\mathfrak{M}_\mathbf{a})) \Leftrightarrow \varphi(\widehat{\mathbf{a}})$ for any $\varphi \in \{\mathbf{Separation}, \mathbf{Uf}, \mathbf{Uniformization}, \mathbf{Reduction}, \mathbf{Enumeration}\}$ and $\neg\mathbf{TotExt}(\mathbb{HF}(\mathfrak{M}_\mathbf{a}))$.

All the notions can be found in [1,2].

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**Dimension function**
**in the theory of closed ordered differential fields**

joint work with Thomas Brihaye and Christian Michaux

The aim of this work is to provide a "well-behaving" dimension function for the theory $CODF$ of closed ordered differential fields introduced, as the model-completion of the theory of ordered differential fields, by M. Singer in 1978 ([1]). For this we adapt usual methods from o-minimality (see for example [3]) and define a notion of $\delta$-cell which leads to a $\delta$-cell decomposition theorem. With these tools, we are able to define a $\delta$-dimension on the class of definable sets in $CODF$ which is a *dimension function* in the sense of the axioms given by L. van den Dries in [3]. We also prove the equivalence between this $\delta$-dimension and two other notions of dimension: the usual differential transcendental degree and the topological dimension associated, in this case, to an appropriate and "natural" topology (it is the trace of the induced topology on the jet-spaces) introduced by C. Michaux in [2].

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GEMMA ROBLES

**Converse Ackermann Property and constructive negation defined with a negation connective**

joint work with José M. Méndez

The Ackermann Property (AP) is a very complex property (see [1], §22.1) predicable of some propositional logics. Roughly speaking, a logic $L$ has the AP if in $L$ necessitive propositions are not derivable from pure non-necessitive ones.
On the other hand, we understand the term "constructive negation" in a broad sense; let us say, for example, that of minimal intuitionistic logic. That is, weak contraposition, weak double negation and weak reductio. Thus, the Converse Ackermann Property (CAP) is the non-derivability of pure non-necessitive propositions from necessitive ones. In our opinion, in logics with a constructive negation, The CAP should be defined as follows: $(A \to B) \to C$ is unprovable if $C$ contains neither $\to$ nor $\neg$. (We note that this definition is far different from the corresponding one in [1], §22.1). We shall define a constructive negation in the logics of [2]. (See also [3]).

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PIET RODENBURG

**Partial operations and the specification of the stack**

It is well known that the traditional method of many-sorted initial algebra specification falls short for the stack data type. Bergstra and Tucker described a variety V of stack algebras, and proved that its initial algebra is not finitely specifiable [1].

The stack signature has two sort names, say 'item' and 'state', one binary operation symbol 'push', and two unary operation symbols, 'top' and 'pop'. Now consider structures in which the interpretations of these operation symbols need not be total. The usual many-sorted stack algebras are among these structures; for example, push(x, y) exists for elements x and y of such an algebra only if x is an item and y is a state. Define a homomorphism to be a function between structures that preserves the sorts and operations. (As far as the operations are concerned, this is just as in Burmeister's survey of partial algebras [2].) Then a simple finite specification exists for the initial element of a class of stack structures the intersection of which with the class of many-sorted algebras is precisely the Bergstra-Tucker variety V.

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Giuseppina Ronzitti

**On the philosophy of intuitionistic mathematics**

The intuitionistic view of mathematics is a very intriguing subject for philosophers as it defies the validity of mathematical ideas, methods and notions which are otherwise considered completely certain or at least are widely taken for granted. Two directions can be identified in the development of intuitionism as a philosophy of mathematics, one emphasizes logico-linguistic analysis and the other focuses on notions that have their origin in idealism. My claim in this talk is that intuitionism, under both characterizations, constitutionally eludes its main philosophical thesis, that mathematics along intuitionistic lines is possible. While this is an example of a philosophical thesis that cannot be proven by means of philosophical arguments (the thesis is proven if intuitionistic theories are actually developed and recognized as mathematics), philosophy could still contribute to it by critically analyzing the basic mathematical concepts and methods of intuitionistic mathematics also in relation to the classical ones, clarifying their ontological and epistemological status. This cannot mean reducing philosophy to logico-linguistical analysis or offering highly intractable subjectivist conceptions as explanations.

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## First-Order Theories of Free Temporal Algebras

This research is devoted to study of first-order theories of free temporal algebras, and continues the investigation of free modal algebras (cf.[2,3]). For any propositional temporal logic $L$, $Var(L)$ is the variety of all temporal algebras sound for $L$ (i.e. $A \in Var(L)$ iff $\forall \varphi \in L$, $A \models (\varphi = 1)$. $F_\gamma(L)$ denotes the free algebra of rank $\gamma$ from the variety $Var(L)$. $Th(F_\gamma(L))$ is the first-order theory of $F_\gamma(L)$; $Th_e(F_\gamma(L))$ is the equational theory of $F_\gamma(L)$; and $Th_q(F_\gamma(L))$ is the quasi-equational theory of $F_\gamma(L)$. Let $T_0$ be the smallest propositional temporal logic (cf. [1]).

**Theorem 1** $Th(F_\omega(T_0))$ is hereditarily undecidable.

Let $T_{K4}$, $T_T$ and $T_{S4}$ are temporal logics obtained from $T_0$ by adjoining axioms saying that the time-flow is transitive, reflexive and both reflexive and transitive (for future and past) respectively.

**Theorem 2** For the free algebras $F_\omega(L)$, where L $= T_{K4}$, $T_T$ or $T_{S4}$, $Th(F_\omega(L))$ are hereditarily undecidable.

Equational theories of all free temporal algebras mentioned above correspond to derivable formulas of mentioned logics which are decidable and hence these equational theories are decidable. Let $T_{intr}$ be the temporal logic generated by all finite frames $\langle F, R\rangle$ with intransitive time-flow relation $R$ (i.e. $\forall a, b, c \in F, (aRb \& aRc) \Rightarrow (b = c \vee c = a)$).

**Theorem 3** $Th(F_\omega(T_{intr}))$ is hereditarily undecidable. $Th_e(F_\omega(T_{intr}))$ is decidable.

The questions whether quasi-equational theories of $Th(F_\omega(T_{intr}))$, $F_\omega(T_0)$, $F_\omega(L)$, for $L = T_{K4}$, $T_T$ or $T_{S4}$, are decidable remain to be open questions.

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Pavel Schreiner

## Automatical recognition of the pretabularity and tabularity in superintuitionistic and positive propositional logics

L. Maksimova in 1972 has proved there exists only three pretabular superintuitionistic propositional logic: Dummet's linear logic $LC$, logic of the second layer $LP_2$ and logic which Hosoi has designated by $LQ_3$.
M. Verkhozina in 1978 has proved similar result for positive logics: there are only two positive pretabular logics ($LC$ and $LP_2$).
Due to the above mentioned results in order to check up when superintuitionistic (positive) propositional logic is pretabular it is enough for us to find out whether it will coincide with one of above mentioned three (two) logics.
Say that a frame is *a fan* if it has a least element and all other elements are maximal; a frame is *a top* if it has a least and a greatest elements, and other elements are incomparable.
Denote by $L + A$ the extension of logic $L$ by an extra axiom scheme $A$.
We proved following statements for logic $L = Int + A(Int^+ + A)$ where $A$ formula with $N$ variables:

*The logic $L$ coincides with logic $LC$ if the formula $A$ is valid on the linearly ordered scale with height $N + 1$ and it is refutable on the three-element fan and four-element top.*
*The logic $L$ coincides with logic $LP_2$ if the formula $A$ is valid on the fan having $2^N$ of the maximal elements and it is refutable on the three-element linearly ordered frame.*
*The logic $L$ coincides with logic $LQ_3$ if the formula $A$ is valid on the top having $2^N$ incomparable elements and it is refutable on the three-element fan and four-element linearly ordered frame.*

Similarly in order to check up when superintuitionistic (positive) propositional logic is tabular it is enough for us to check up whether the formula $A$ is refutable on three (two) scales: linear, fan and top (linear scale and fan).
The author creates the programs realizing above described algorithms.

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## The projective spectrum as a distributive lattice

joint work with Thierry Coquand and Henri Lombardi

The spectrum of a commutative ring is the collection of prime ideals endowed with the Zariski topology. To arrive at a point–free description of this topological space, one can define the corresponding distributive lattice directly from the ring, without any talk of prime ideals. This idea, which presumably is due to Joyal [1], has recently been reactivated to start a first–order treatment of commutative algebra (see, for instance, [2]).

By continuing this road, we bring the projective spectrum of a graded ring into the point–free realm. More specifically, we present a distributive lattice whose prime filters correspond to the homogeneous prime ideals of the graded ring. This also is the prime example of a non–affine scheme in the context of formal topology, and turns out to be a particular case of a fairly general glueing method for finitely many distributive lattices.

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MARINA SEMENOVA

## Embedding lattices into convex geometries

*A convex geometry* is a closure space $(X, \varphi)$ with the anti-exchange property and such that $\varphi(\varnothing) = \varnothing$.
We consider lattices embeddable into classes of closure lattices of convex geometries of a particular type. Among those, are lattices of convex subsets of posets and ones of vector spaces, suborder latices, and subsemilattice lattices. In particular, we prove that, for any of the classes mentioned above, the class of sublattices forms a (finitely based) variety. We also investigate the possibility to embed so-called *lower bounded* lattices into closure lattices of particular finite convex geometries. A big part of the mentioned results are obtained in collaboration with Friedrich Wehrung. The corresponding references are as follows:

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DMITRIJ SKVORTSOV

## An Extension of Quantified Dummett's Logic that is Complete with respect to Kripke Sheaves

We consider superintuitionistic predicate logics (without equality and functional symbols), i.e., extensions of intuitionistic predicate logic **QH** closed under modus ponens, universalization and predicate substitution. We consider the standard predicate Kripke semantics **K** and its generalization, the Kripke semantics with equality **KE** or equivalently, the semantics of Kripke sheaves [3].
Let us consider the following formulas:

$E$ : $\neg\neg\exists x P(x) \to \exists x \neg\neg P(x)$ (a version of Markov's Principle) ;
$Z$ : $(P \to Q) \vee (Q \to P)$ ;
$D$ : $\forall x(P(x) \vee Q) \to \forall x P(x) \vee Q$ ;
$D^-$ : $\forall x(\neg P(x) \vee Q) \to \forall x \neg P(x) \vee Q$ ;
$D^*$ : $\forall x(P(x) \vee Q) \to Q \vee \forall x \exists y (P(y) \& \neg\neg[R(x,x) \to R(x,y)])$ .

The predicate version of Dummett's logic $\mathbf{QLC} = [\mathbf{QH} + Z]$ is **K**-complete [1]. On the other hand, $[\mathbf{QH}+E]$ is **K**-incomplete [2]; similarly, $[\mathbf{QLC}+E]$ is **K**-incomplete and its **K**-completion is $[\mathbf{QLC}+D]$.

**Theorem**
(1) $D^*$ is valid in any Kripke sheaf validating $E$, but $[\mathbf{QLC}+E] \nvdash D^*$;
(2) the logic $[\mathbf{QLC}+D^*]$ is **K**-incomplete, but **KE**-complete.

So, $[\mathbf{QLC}+E]$ is **KE**-incomplete and its **KE**-completion is $[\mathbf{QLC}+D^*]$.
Note that $[\mathbf{QLC}+E] = [\mathbf{QLC}+D^-] \subset [\mathbf{QLC}+D^*] \subset [\mathbf{QLC}+D]$.

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Alexey I. Stukachev

## On structures decidable in admissible sets

For arbitrary structure $\mathfrak{M}$ of computable signature $\sigma$ and admissible set $\mathbb{A}$ with $M \subseteq U(\mathbb{A})$ we call $\mathfrak{M}$ *decidable in* $\mathbb{A}$ if

$$\{\langle\varphi, \bar{m}\rangle | \varphi \in F_\sigma, \bar{m} \in M^{<\omega}, \mathfrak{M} \models \varphi(\bar{m})\}$$

is $\Sigma$-subset of $\mathbb{A}$. In the same way notions of computable and $n$-decidable structures in $\mathbb{A}$ are defined. We show that if $\mathfrak{M}$ is 1-decidable in $\mathbb{HF}(\mathfrak{M})$ when $\mathbb{HF}(\mathfrak{M})$ has universal $\Sigma$-function and possess reduction principle, but not always uniformization property. We prove

**Theorem 1.** Let $\mathfrak{M}$ be 1-decidable in $\mathbb{HF}(\mathfrak{M})$. Then $\mathbb{HF}(\mathfrak{M})$ has uniformization property iff some existential Skolem extension of $\mathfrak{M}$ is computable in $\mathbb{HF}(\mathfrak{M})$.

We describe decidable in HF-superstructures computable structures, generalising Nurtazin's theorem from [1].

**Theorem 2.** For computable structure $\mathfrak{M}$ and $n \leqslant \omega$ the following are equivalent:
1) $\mathfrak{M}$ is $n$-decidable in $\mathbb{HF}(\mathfrak{M})$;
2) for any $\mathfrak{N} \simeq \mathfrak{M}$ $\mathfrak{N}$ is $n$-decidable relative to $D(\mathfrak{N})$;
3) for any computable $\mathfrak{N} \simeq \mathfrak{M}$ $\mathfrak{N}$ is $n$-decidable.

**Theorem 3.** Linear order $\mathfrak{L}$ is 1-decidable in $\mathbb{HF}(\mathfrak{L})$ iff $\mathfrak{L}$ is a sum of finite number of dense linear orders and finite linear orders.

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Dariusz Surowik

**The different methods of rejection of the thesis of determinism in temporal logic systems**

Usually arguments on determinism are based on *the principle of causality, the principle of bivalency* or *the tertium non datur*. To create an indeterministic logic system we have to reject one of these principles. Determination should be considered in a temporal context, then we will concentrate our attention on temporal logic systems. There are different methods of construction of an indeterministic temporal logic. In our paper we would like to discuss the following solutions:

- Many-valued temporal logic,
- Branching time temporal logic,
- Temporal logic with modal-tense operators,
- Intuitionistic temporal logic.

We will show advantages and defects of discussed systems. Moreover, we will show that some expressions of the thesis of determinism in a temporal logic language are true in some of the considered systems. The thesis of determinism in a language of temporal logic is formulated as $F\alpha \vee F\neg\alpha$ (argument based on *the tertium non datur*) or $\alpha \to HF\alpha$ (argument based on *the principle of causality*). In our considerations we will show the conditions, what should be satisfied by semantic of the temporal logic, if the the thesis of determinism formulated as above should be true. We will give an answer for the following question: Is possible a construction of an indeterministic temporal logic satisfying the following conditions:

- a language is a proper formal tool for describe indeterministic world,
- there are two logical values,
- there are no condition upon structure of time,
- the are no specific operators apart from standard temporal operators.

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Boža Tasić

## $HSP \neq SHPS$ for Commutative Rings with Identity

joint work with John Lawrence

Let $I$, $H$, $S$, $P$, $P_s$ be the usual operators on classes of rings: $I$ and $H$ for isomorphic and homomorphic images of rings and $S$, $P$, $P_s$ respectively for subrings, direct, and subdirect products of rings. If $\mathcal{K}$ is a class of commutative rings with identity (and in general of any kind of algebraic structures) then the class $HSP(\mathcal{K})$ is known to be the variety generated by the class $\mathcal{K}$. Although the class $SHPS(\mathcal{K})$ is in general a proper subclass of the class $HSP(\mathcal{K})$ for many familiar varieties $HSP(\mathcal{K}) = SHPS(\mathcal{K})$. Our goal is to give an example of a class $\mathcal{K}$ of commutative rings with identity such that $HSP(\mathcal{K}) \neq SHPS(\mathcal{K})$. As a consequence we will describe the structure of two partially ordered monoids of operators.

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Laura Tesconi

**A Strong Normalization Theorem for natural deduction with general elimination rules**

The aim of the present work is to prove a strong normalization theorem for a system of natural deduction with general elimination rules, as first presented in [J. von Plato, Natural deduction with general elimination rules. *AML*, 40, 2001]. The main feature of this system concerns the treatment of the elimination rules, which differ from the usual ones in that they are all formulated in the manner which is usual for disjunction elimination, with an arbitrary consequence (these rules have been called *general elimination rules* because the usual elimination rules for conjunction and implication come out as special cases).
The line of reasoning of the proof bears similarities to a strong normalization proof for a term assignment system proposed in [F. Joachimski and R. Matthes, Short proofs of normalization for the simply typed $\lambda$-calculus, permutative conversions and Gödel's T. *AML*, 42, 2003] but it results specially suited for the treatment of derivation trees.
The idea behind the proof is that of showing that every derivation belongs to the set of strongly normalizing derivations. So, a preliminary step consists in giving an appropriate characterization of this set. Then, suitable simple sufficient conditions for a derivation $\mathcal{D}$ to belong to the set of strongly normalizing derivations are established. These conditions will be proposed according to the structure of $\mathcal{D}$, which ranges over five possible kinds, and will concern either strong normalizability of certain subderivations of $\mathcal{D}$ or of a specific immediate reduct $\mathcal{D}^*$ of $\mathcal{D}$. The main result of this stage will consist in the proof that strong normalizability of $\mathcal{D}$ is entailed by strong normalizability of $\mathcal{D}^*$. Now, a couple of preliminary lemmas are proved that specify which kind of syntactic manipulations can be performed on strongly normalizing derivations, while preserving strong normalizability. To this extent, a suitable "measure" of complexity of derivations must be defined, to play the role of an induction parameter. Finally, the Strong Normalization Theorem results as an immediate consequence of the lemmas.

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Nikolaus Thiel

**Mahlo & Veblen**

joint work with Ben Gibbons

Being inspired by the work of Hausdorff, Mahlo presents in 1911 two hierarchies of regular cardinals: the $\pi_\alpha$-numbers and the $\rho_\alpha$-numbers; in modern terms: $\alpha$-weakly inaccessible and "$\rho$-weakly Mahlo" cardinals, respectively .
In 1908, Veblen answered partly Hardy's question "How to associate with each countable limit ordinal a unique fundamental sequence" by defining the Veblen functions.
In my talk I will connect Mahlo's hierarchies to Veblen's. For this end I will present the constructive analogues of inaccessibility and mahloness in CZF and determine the proof-theoretic strength of some CZF-extensions with existence axioms for set-inaccessibles and set-Mahlos. The resulting ordinal plays an important role in the Veblen hierarchy.
This work is a result of my PhD-thesis and the PhD done by Ben Gibbons. Both doctorates have been submitted at the University of Leeds (UK) under the supervision of Michael Rathjen.

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Carlo Toffalori

**Superdecomposable pure injective modules over integral group rings**

joint work with Gena Puninski and Vera Puninskaya

A remarkable peculiarity in the analysis of pure injective modules over a given ring $R$ is the existence of "superdecomposable" objects, i.e., pure-injective $R$-modules without any indecomposable nonzero direct summand. Not so much is known about the existence of these superdecomposables. Ziegler [3] related this problem and the width of the ring $R$ and showed that, if the lattice of pp-formulas over $R$ has width, then no superdecomposable pure injective $R$-module exists; for $R$ countable, the converse is also true.
Referring to this criterion, G. Puninski proved in [1] that superdecomposable pure injective modules always exist over arbitrary non-domestic string algebras over countable fields. Here we investigate this question for group rings $R = DG$, where $D$ is a Dedekind domain of characteristic zero (in particular $D$ is the ring $\mathbf{Z}$ of integers) and $G$ is a nontrivial finite group. We show that every integral group ring $\mathbf{Z}G$ admits superdecomposable pure-injective modules. In detail, first we see that this claim is true for every $G$ provided that it is valid for every group of prime order. Then we show existence of superdecomposable pure injective modules in this restricted setting. The crucial fact here is that $\mathbf{Z}C(p)$ has the structure of a pullback of Dedekind domains; indeed our existence result is quite general and applies to arbitrary pullbacks of Dedekind domains (which are not fields) over a common residue field. Our approach follows ideas from [1].

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Tero Tulenheimo

## Independence-Friendly Modal Logic

Independence-friendly modal logic (IFML) is obtained from standard modal logic (ML) by allowing diamonds (boxes) to be logically independent from any syntactically superordinate boxes (diamonds). A survey is given about the known properties of IFML.
The semantics of IFML is formulated in game-theoretical terms, making use of the notion of uniform strategy. It is shown that IFML has greater expressive power than ML relative to arbitrary modal structures, and can be translated into the 3-variable fragment of first-order logic. Corresponding results hold of tense logic. When restricted to a simple tense-logical setting (evaluation over strict linear orders), the expressive power of IF tense logic however coincides with standard tense logic. The syntax of IFML can be modified to allow independence of diamonds (boxes) from conjunctions (disjunctions). It is shown that the resulting extended IF modal logic can no longer be translated into first-order logic. Finally, the satisfiability problem of IFML is shown to be decidable, by proving IFML to possess the strong finite model property.

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LUIS ADRIAN URTUBEY

## Vagueness, Logic and Information Processing

Contemporary approaches to vagueness mostly draw on two salient features of vague predicates, viz. the existence of the so called "boundary cases" and the Sorites contradiction. Although some theories are concerned also with the metaphysical problem of vagueness, most of them have paid attention above all to vagueness as a feature of claims and representations.
In a sorites sequence:

i) initially there are true predications (with each predecessor being also true),
ii) later there are false predications (with each successor being false as well),
iii) there is no determinate fact of the matter about the transition from true predications to false ones.

As many authors have noted, if one considers what it would take to fully accomodate boundarylessness, it seems that for the successive statements in a sorites sequence, there are semantic requirements in play that cannot be simultaneously satisfied.
There are different proposals concerning the way to avercome the logical incoherence that stems from this lack of boundaries. Psychologistic approaches to the semantics of vague predicates have focused on the psychology of the application of vague words and envision judgments like those in the sorites series as the activation of a psychological category. Accordingly, psychological treatments allow to place the problem in relation with the setting of cognitive sciences, having shown it has to do with information processing in a particular system. In the same vein Barwise and Sellingman's "Information Flow" suggested an information-theoretic line of research into vagueness.
My aim in this contribution is to make some considerations with respect to the interpretation of psychological semantics of vagueness from the standpoint of an information-theoretic analysis by taking into account this overall framework.

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ALEX USVYATSOV

**Categoricity in complete metric spaces**

Generalizing Henson and Iovino's logic for normed spaces, we consider positive theories of metric spaces. Our notion of embedding is existence of $\epsilon$-embedding for all positive $\epsilon$ (which is weaker than the regular notion, i.e. isometry, and seems more suitable for classes of analytic structures, although still not completely satisfactory). We therefore deal with topological versions of $\lambda$-stability. While general stability for a monster metric space happens to be a particular case of stability for homogeneous models (so methods of homogeneous model theory apply), notions of $\omega$-stability and superstability are quite different. We present the beginning of the theory and study categorical classes of metric positive theories.
This work is a part of my PhD thesis which is done under the supervision of Prof. S. Shelah.

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Yde Venema

**McNeille completions of lattice expansions**

The MacNeille completion of a lattice $L$ is a certain complete extension of $L$, preserving all meets and joins of $L$. In the talk we introduce two ways to extend an arbitrary operation on a lattice to an operation on the MacNeille completion, thus arriving at the concept of a lower and an upper MacNeille completion of a lattice expansion.
In the talk I will review some results concerning MacNeille completions which have been obtained over the last couple of years by me and various (co-)authors, including Bezhanishvili, Gehrke, Givant, Harding, and Theunissen. The emphasis will be on the question which equational properties of a lattice expansion are McNeille-canonical, that is, preserved under moving to the MacNeille completion. The construction generally has a bad reputation in this respect, with the distributive law of the lattice itself as a crown witness. However, we will see that under some mild condition on the (extra) operations, quite a few positive results are possible. Finally, apart from studying MacNeille completions in isolation, I will also compare them to other ways of completing lattice expansions. A main result that I will mention is that all McNeille-canonical varieties of lattice expansions are also canonical in the ordinary sense, i.e., closed under taking canonical extensions.

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Jan von Plato

## Consistency of arithmetic through sequent calculus in natural deduction style

In sequent calculus in natural deduction style, any multiplicities of active formulas can appear in the premisses of logical rules. No structural rules are needed, because multiplicity zero corresponds to weakening and greater than one to contraction.
Sequent calculus in natural deduction style is applied to simplify and complete Gentzen's third (1938) consistency proof of arithmetic.

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V. Vuksanovic

**A canonical partition theorem for trees**

We prove that for every leafless downwards closed subtree $T$ of a regular tree $k^{<\mathbb{N}}$ without terminal nodes and for every finite weakly embedded subtree $A$ of $T$, there is a finite list of equivalence relations $\{\mathbb{E}_i : 1 \leqslant i \leqslant T(A)\}$ on $Em^A(T)$, where $Em^A(T)$ is the set of all finite weakly embedded subtrees of $T$ which have the same embedding type as $A$, such that for every other equivalence relation $\mathbb{E}$ on $Em^A(T)$ there is a strongly embedded subtree $S$ of $T$ of height $\omega$ and an equivalence relation $\mathbb{E}_i$ from the list such that

$$\mathbb{E} \restriction Em^A(S) \;=\; \mathbb{E}_i \restriction Em^A(S).$$

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Agatha C. Walczak-Typke

## Algebraic Structures in Set Theory without the Axiom of Choice

In set theory without the Axiom of Choice, there exist infinite sets $X$ such that $\aleph_0 \not\leqslant |x|$. Such sets are said to be *Dedekind-finite*. Sets of this kind have been studied in, for example, [1] and [2]. We explore the similarities and differences between sets of Dedekind-finite cardinalities that admit an algebraic structure and certain classes of first-order theories and models . These explorations have as their starting point the analogies between strongly minimal structures and amorphous sets (sets that have only finite and co-finite subsets), first noted in [3]. We employ both model-theoretic and forcing methods.

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Andreas Weiermann

## Does (concrete) mathematics matter to Gödel's theorem?

In 1982 Gina Kolata reported in Science [2] ("Does Gödel's theorem matter to mathematics") about two spectacular results highlighting the mathematical relevance of the Gödel incompleteness theorems about the standard axioms of the natural numbers. In our talk we report on some recent progress about classifying the threshold transition from provability to unprovability in these two examples and we will provide an attractive and generally accessible description of certain transfinite ordinals which are related to unprovability issues. We will indicate that this advance is based on a surprising cross-fertilization between concrete mathematics (or analytic combinatorics), which finds its main applications in the efficiency analysis of computer algorithms and mathematical logic, and we expect that this will lead to a long term interdisciplinary synergy between different communities.

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Roman Wencel

**A genaralization of o-minimality for expansions of Boolean algebras**

$(M, \leqslant, \ldots)$, a partially ordered $L$-structure is called o-minimal iff every $L$-definable subset of $M$ is a finite Boolean combination of sets defined by formulas of the form $x \leqslant a$ and $x \geqslant b$, where $a, b \in M$. This definition was introduced by Toffalori in 1998 and investigated in case of structures with partial orderings derived from Boolean algebras (Toffalori, Leonesi, Newelski and myself). The assumption that an expansion of a Boolean algebra is o-minimal implies that the underlying Boolean algebra has finitely many atoms. Here we propose the following definition, which coincides with the Toffalori's notion of o-minimality in case of Boolean algebras with finitely many atoms.

**Definition** $(B, \ldots)$*, an expansion of a Boolean algebra* $B$ *to the language* $L$ *is o-minimal iff every* $L$*-definable subset of* $B$ *is definable in the language of pure Boolean algebras. We say that the theory* $T = Th(B, \ldots)$ *is o-minimal iff every model of* $T$ *is o-minimal.*

We investigate this notion in case of expansions of Boolean algebras admitting weak elimination of imaginaries. In this context we discuss the following problems

- weak orthogonality of certain types,
- exitence and uniqueness of prime models over sets,
- Vaught conjecture,
- o-minimality of a structure and its theory.

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Gunnar Wilken

**Characterizing Closure Properties of Ordinals**

We will illustrate the exact correspondence between ordinal notations derived from Skolem hull operators and descriptions of ordinals in terms of $\Sigma_1$ elementarity, an approach developed by T. J. Carlson. Using appropriate means of relativization on both sides it is possible to give mutual characterizations of ordinals whose closure properties do not exceed the closure properties of the proof theoretic ordinal of the fragment $\Pi^1_1$-$CA_0$ of second order number theory.

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Teruyuki Yorioka

## Gap spectra under Martin's Axiom

For subsets $\mathcal{A}$ and $\mathcal{B}$ of $\mathcal{P}(\omega)$, a pair $\langle \mathcal{A}, \mathcal{B} \rangle$ is called a pregap if $a \cap b$ is finite for every $a \in \mathcal{A}$ and $b \in \mathcal{B}$. A pair $\langle \mathcal{A}, \mathcal{B} \rangle$ is called a gap if it forms a pregap and there is no interpolation, i.e. there exists no $c \in \mathcal{P}(\omega)$ such that for all $a \in \mathcal{A}$, $a$ is almost included in $c$ and for all $b \in \mathcal{B}$, $b \cap c$ is finite. If the order types of $(\mathcal{A}, \subseteq/\text{finite})$ and $(\mathcal{B}, \subseteq/\text{finite})$ are $\kappa$ and $\lambda$ respectively, then we call a pregap $\langle \mathcal{A}, \mathcal{B} \rangle$ a $(\kappa, \lambda)$-pregap, and a gap $\langle \mathcal{A}, \mathcal{B} \rangle$ a $(\kappa, \lambda)$-gap.

It follows from ZFC that there exist some types of gaps: $(\omega_1, \omega_1)$-gaps and $(\omega, \mathfrak{b})$-gaps (where $\mathfrak{b}$ is the smallest size of an unbounded family). (These are due to Hausdorff and Rothberger.) If $(\kappa, \lambda) \neq (\omega_1, \omega_1)$, then a forcing notion adding an interpolation of a $(\kappa, \lambda)$-gap by finite approximations has the countable chain condition. So Martin's Axiom forbids such short gaps, i.e. if there exists a $(\kappa, \lambda)$-gap under Martin's Axiom, then either $\kappa$ or $\lambda$ is equal to the continuum or both $\kappa$ and $\lambda$ are equal to $\omega_1$. Kunen had proved that the following statements are consistent with ZFC:

- Martin's Axiom + there are $(\mathfrak{c}, \mathfrak{c})$-gaps and $(\omega_1, \mathfrak{c})$-gaps.
- Martin's Axiom + there are no $(\mathfrak{c}, \mathfrak{c})$-gaps and no $(\omega_1, \mathfrak{c})$-gaps.

And there have been open questions whether or not the following statements are consistent with ZFC (These problems are in Marion Scheepers' article *Gaps in* $\omega^\omega$) :

(1) Martin's Axiom + there are $(\mathfrak{c}, \mathfrak{c})$-gaps but no $(\omega_1, \mathfrak{c})$-gaps.
(2) Martin's Axiom + there are $(\omega_1, \mathfrak{c})$-gaps but no $(\mathfrak{c}, \mathfrak{c})$-gaps.

I give a sketch of the proof of the consistency of statement 1. This proof uses *forcing notions with models as side conditions*, a method due to Stevo Todorcevic. The consistency of statement 2 is still open.

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Richard Zach

## Gödel's First Incompleteness Theorem and Detlefsen's Hilbertian Instrumentalism

Detlefsen [1, 2, 3] has provided a sustained defense of an instrumentalist account of mathematics along the lines of Hilbert's Program. Specifically, he has defended this view against Gödel's First (G1) and Second (G2) Incompleteness Theorems, which are widely taken to "destroy" Hilbert's Program. In [2], Detlefsen has argued that challenges based on G1 such as those put forward by Kreisel, Prawitz, and Smoryński are ineffective against Hilbertian Instrumentalism. I argue that Detlefsen can escape the argument from G1 only by evading some crucial questions regarding the nature of the gain in "efficiency" of ideal methods. The restrictions on instrumentalist mathematics necessary to avoid the challenge posed by G1 also result in an implausible account of mathematical proof.

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Ernst Zimmermann

**Natural Deduction with Single Assumptions**

Natural Deduction is well known to have no explicit structural rules - in contrast to calculi of sequents. But Natural Deduction has implicit structural rules, hidden in assumption discharging rules, discharging arbitrary classes of assumptions. The paper undertakes the task to decompose assumption discharging rules by introducing repetition rules: one repetition rule for implication, one for disjunction and one for existence. Repetition rules allow natural deduction to work with single assumptions, i.e. to discharge with one rule application at most one single assumption, and to remain nevertheless in the realm of intuitionistic and classical logic; and repetition rules allow to give a certain linearisation of intuitionistic and classical logic within Natural Deduction. The paper proofs within a framework of introduction, elimination and repetition rules various normalisation properties for intuitionistic and for classical predicate logic up to strong normalisation.

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ENRICO ZOLI

**On Schmidt's $\sigma$-ideal**

$\sigma$-ideals defined in the Cantor space (or in the Baire space) associated to certain games between two players have been investigated by many authors (here I limit myself to mentioning some members of the Polish school: Mycielski, Rosłanowski, Bartoszyński, Balcerzak and Solecki).
My contribution deals with the so-called $(\alpha, \beta)$-games – defined in the real line – introduced by the number-theorist Wolfgang Schmidt in 1966. Precisely, my aim is that of presenting the set-theoretical properties of the $\sigma$-ideal consisting of the "absolutely losing" (in Schmidt's sense) subsets of $\mathbf{R}$. This ideal turns out to be orthogonal to the $\sigma$-ideal made up of the sets being *both* meager and null in the sense of Lebesgue. Moreover, Schmidt's $\sigma$-ideal (better: a suitable slight variant of it) turns out to be coanalytic-generated.
Some consequences of the two facts above are then discussed, among which the existence of very pathological sets *a la* Mahlo-Luzin-Sierpinski-Rothberger, i.e., constructed by combining transfinite recursions and the assumption of CH.
Finally, I list a number of related open problems: they all appear to rely on infinitary combinatorics.
My communication is essentially drawn out from the paper "On the equivalence between CH and the existence of certain $\mathcal{I}$-Luzin subsets of $\mathbf{R}$", very recently appeared on the Georgian Mathematical Journal.

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# Contributed Papers Submitted by Title

**Adib Ben Jebara,** ***An axiom to settle the continuum hypothesis?***

**Izabela Bondecka-Krzykowska,** ***The four-color theorem and its philosophical consequences***

**John Corcoran,** ***Boole's Solutions Fallacy***

**Ruy J. G. B. de Queiroz,** ***Towards normalization for proof-graphs: basic transformations***

**Jan Gałuszka,** ***Classes of totally commutative and idempotent groupoids***

**Petr Hájek,** ***On arithmetic in the Cantor-Łukasiewicz fuzzy set theory***

**Zurab Janelidze,** ***Mal'tsev varieties and subtractive categories***

**Andrei Kouznetsov,** ***Deduction chains and DCL algorithm for guarded logic***

**Barbara Majcher-Iwanow,** $G_\delta$***-companions of Orbits of Polish Group Actions***

**Larisa Maksimova,** ***Interpolation and Projective Beth Property in Extensions of Minimal Logic***

**Veta Murzina,** ***The modal logic based on strictly ordered*** $\alpha$***-spaces***

**Wim Ruitenburg,** ***More Intuitionistic Theories with Quantifier Elimination***

**Denis I. Saveliev,** ***Order properties of*** $\kappa$***-dense, Borel, and other sets of reals***

**Levan Uridia,** ***Lattice of varieties of weak monadic algebras***

**Guohua Wu,** ***1-Generic Splittings of Computably Enumerable Degrees***

**V. K. Zakharov,** *Descriptions of all Tarski sets*
*and all standard supertransitive models of ZF*

Adib Ben Jebara

**An axiom to settle the continuum hypothesis?**

Paul Cohen used a set of generic reals to prove the consistency of the negation of the continuum hypothesis with other axioms. It is my opinion that such sets do not really exist for a Platonist. My opinion is that the continuum hypothesis is true.

Here is a tentative axiom from me to try to prove it. Axiom: An infinite subset of the power set of $N$ has a bijection either with a countable union of (pair wise disjoint) sets of n elements or with a countable Cartesian products of (pair wise disjoint) sets of n elements.

Andreas Blass proved that this axiom is equivalent to the continuum hypothesis. So, the axiom is consistent with the other usual axioms and independent from them, from the works of Kurt Gödel and Paul Cohen, respectively. Andreas Blass used the assumption that the Cartesian product is not the empty set but he did not use the axiom of choice.

The question which remains is: is it a good axiom? My opinion is that it is realistic for a Platonist. But may be the axiom is not simple enough and may be a simpler one could be found.

In Logic Colloquium 2001, "All things are numbers", I wrote that the numbers of attributes of God could be the alephs, God being the set of all laws of nature and of ethics. I consider $\aleph_0$ to be the number of attributes of God enabling life and $\aleph_1$ the number of attributes of God enabling the immortality of souls. I consider that there is no such thing as immortal bodies of people in Heaven. Which is a possible philosophical interpretation of the continuum hypothesis.

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Izabela Bondecka-Krzykowska

## The four-color theorem and its philosophical consequences

The four-color theorem is An example of mathematical truth which posses only computer-assisted proof. On the example of this theorem we discuss some philosophical connected with admitting computer proofs in mathematics, in particular the status of mathematical knowledge as a pattern of science whose truths are known a priori.

In the paper ways in which computers are applied in mathematics are considered, too. Incorporating computers to mathematics, especially in proving theorems, reveals many philosophical problems, such as the status of mathematical truths (distinction between a priori and a posteriori), the status of mathematics as a standard for formal science and finally the problem of admissible methods of "doing" mathematics. Most of these problems remain unsolved because theirs solutions very often depend on interpretations of such terms as a mathematical proof, a priori or a posteriori.

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John Corcoran

**Boole's Solutions Fallacy**

Boole's Solutions Fallacy is the mistake of using "being a solution to an equation" as a sufficient condition for "being a consequence of it". Let us say that an *$x$-equation* is an equation having $x$ as its only unknown: $x(x-1)=0$, $xx=x$, $x=0$ and $x=1$ are $x$-equations. Let us say that an *L$x$-equation* (or *left-$x$-equation*) is an equation having $x$ as its left term: $x=x$, $x=(1-x)$, $x=0$ and $x=1$ are L$x$-equations, but $x(x-1)=0$ and $xx=x$ are not. In Boole's logical algebra or algebraic logic, every L$x$-equation that is a consequence of a given $x$-equation is a solution to it: $x=0$ is an L$x$-equation that is a consequence of $xx=0$ and is a solution, $00=0$. However, not every L$x$-equation that is a solution to a given $x$-equation is a consequence of it; $x=(1-x)$, $x=0$ and $x=1$ are L$x$-equations that are all solutions to $x(x-1)=0$, but none are consequences. Boole never committed the fallacy in cases as simple as this. Nevertheless, as this paper shows, this fallacy is involved in many of Boole's trains of thought. Boole's confusion of solutions of equations with deductions from equations was so compete that he even spoke of deducing solutions. This fallacy is an integral part of one of Boole's proudest "discoveries" — that logic is a form of analysis or algebra. Boole thought that he had finally determined the true nature of logical deduction; he thought he discovered that deducing a consequence from given premises is really solving a system of equations. This paper supplements my "Aristotle's *Prior Analytics and Boole's Laws of Thought*", *History and Philosophy of Logic 24* (2003) 261–288.

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RUY J. G. B. DE QUEIROZ

**Towards normalization for proof-graphs: basic transformations**

joint work with Gleifer Vaz Alves and Anjolina Grisi Oliveira

Here we wish to report on the first steps towards obtaining normalization for N-Graphs, showing the so-called **basic transformations**.

The N-Graphs proof system defined by de Oliveira with the intention of developing of a calculus which uses classical logic as starting point but also imports geometric and graph-theoretic techniques from linear logic. N-Graphs features are: a multiple conclusion ND augmented with structural rules, plus techniques from proof-nets such as, e.g., splitting and sequentialization [1]. The result is a refinement of Kneale's (and Shoesmith–Smiley's) symmetric ND recast in a "geometric" perspective. Some of the transformations for N-Graphs were motivated by Ungar's multiple conclusion ND calculus [2].

The basic transformations are divided into two sets, the *logical* transformations and the *structural* transformations. In the first set we have two basic logical reductions, i.e. $\beta$- and $\eta$- reductions. The second set has different sorts of structural reduction rules. The first subset has some *basic reductions*. A second subset deals with the *thinning reductions* and it is somewhat based on Ungar's approach. The last subset handles *expansion* and *contraction* reductions.

We expect that a normalization theorem for N-Graphs will help bringing useful ingredients into currently open questions such as, e.g., identity of proofs, and the correspondence between proofs and processes.

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Jan Gałuszka

## Classes of totally commutative and idempotent groupoids

Let $L$ be a first order language of the groupoid theory and let $t$ be a term over $L$. For a given groupoid $\mathfrak{G}$ symbol $t^{\mathfrak{G}}$ denotes the interpretation of the term $t$ in $\mathfrak{G}$. We say that the identity $t_1 = t_2$ over $L$ is $\mathfrak{G}$-*regular* if and only if there exist terms $s_1$ and $s_2$ such that $s_1^{\mathfrak{G}} = t_1^{\mathfrak{G}}$, $s_2^{\mathfrak{G}} = t_2^{\mathfrak{G}}$ and the identity $s_1 = s_2$ is regular where the term *regular* means that the same variables appear on both sides of this identity. Analogously, we say that the identity $t_1 = t_2$ is $\mathfrak{G}$-*nonbalanced* if and only if $s_1^{\mathfrak{G}} = t_1^{\mathfrak{G}}$, $s_2^{\mathfrak{G}} = t_2^{\mathfrak{G}}$ for some terms $s_1$ and $s_2$ where $s_1$, $s_2$ are such that the identity $s_1 = s_2$ is nonbalanced (i.e. this identity is not balanced where the term *balanced* means that each variable occurs equally often in $s_1$ and $s_2$). Below a regular (resp. nonbalanced) term denotes a $\mathfrak{G}$-regular (resp. $\mathfrak{G}$-nonbalanced) term for a given groupoid $\mathfrak{G}$.

A groupoid $\mathfrak{G}$ is *totally commutative* if each its essentially binary term operation is commutative (see [3]). The class of all idempotent and totally commutative groupoids falls into exactly two classes: the variety of Steiner quasigroups formed by groupoids satisfying nonregular identities and the variety of groupoids satisfying the identity $(xy)y = (yx)x$ (see [2]). Among others it is proved that the class of all idempotent totally commutative regular groupoids satisfying binary and nonbalanced identities is a nonaxiomatized proper subclass of the class of all idempotent totally commutative regular groupoids. To prove this result some theorems of model theory (see [1]) and construction presented in [4] are used.

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Petr Hájek

## On arithmetic in the Cantor-Łukasiewicz fuzzy set theory

Axiomatic set theory with full comprehension is known to be consistent in Łukasiewicz infinitely-valued fuzzy predicate logic, as proved by White [3]. It remains consistent if we add a constant $\omega$ (for natural number) satisfying $x \in \omega \equiv (x = \emptyset \vee (\exists y \in \omega)(x = \{y\})$ and induction schema

$$(\varphi(\emptyset)\&(\forall x) \in \omega)(\varphi(x) \equiv \varphi(\{x\})) \rightarrow (\forall x \in \omega)\varphi(x)$$

for formulas $\varphi$ *not containing the constant* $\omega$. If we assume this induction schema for *all* formulas $\varphi$ then we can define addition and multiplication of natural numbers and prove all axioms of Peano arithmetic, which seems pleasing. But we can prove too much we can define truth for natural numbers which is self-referring and commutes with connectives, which leads to a contradiction in the style of [1]; our set theory turns out to be inconsistent. Details will be published as [2].

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Zurab Janelidze

## Mal'tsev varieties and subtractive categories

A class $C$ of categories is said to be a localization of a class $V$ of varieties of universal algebras, if, a variety $W$ is in $V$ whenever for any algebra $A \in W$ the category of all triples $(B, f : B \to A, g : A \to B)$, where $B$ is an algebra in $W$ and $f$, $g$ are morphisms of algebras such that $fg = Id(A)$, belongs to $C$. As it was shown in [B] the class of unital categories (which generalize Jónsson-Tarski varieties [JT]) is a localization of the class of Mal'tsev varieties in the sense of J. D. H. Smith [S]. I introduce a notion of a subtractive category, which generalizes the notion of a subtractive variety in the sense of A. Ursini [U], and I prove that the class of subtractive categories is also a localization of the class of Mal'tsev varieties. Having this at hand, one is able to transform theorems for subtractive categories to theorems for Mal'tsev varieties. In particular, I show how several well-known characterizations of Mal'tsev varieties can be derived from the "categorical versions" of the characterizations of subtractive varieties given in [AU].

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Andrei Kouznetsov

**Deduction chains and DCL algorithm for guarded logic**

The notion of deduction chain (DC) was suggested by K.Schütte to prove the completeness of first order logic (FOL). There is a question whether it is possible to provide the decision procedure for the guarded fragment of FOL (GF) on the basis of DC-rules. We propose here the decision procedure, or DCL algorithm, for the GF of FOL which rules are close to the ones of DC. DCL algorithm can be viewed as a "mirrow" of the tableau algorithm with the use of the so called blocking technique.

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Barbara Majcher-Iwanow

## $G_\delta$-companions of Orbits of Polish Group Actions

Let $G$ be a closed subgroup of the group $S_\infty$ of all permutations of $\omega$. Let $(X, G)$ be a Polish $G$-space and $\tau$ be the corresponding topology. In [B] H.Becker considers finer topologies $t$ on $X$ which in some sense correspond to fragments of $L_{\omega_1,\omega}$ in the case of the $S_\infty$-space of the logic action. Such topologies are called *nice* and they are defined by *nice bases* $\mathcal{B}$, which are countable families of $\tau$-Borel subsets (the formal definitions can be found in [B]).

Let $X_0$ be an invariant subset of $X$. A $\tau$-$G_\delta$-subset $X_1$ is called a *companion* of $X_0$ if $X_0$ and $X_1$ have the same $\tau$-closure and any $\tau$-open member of $\mathcal{B}$ is $\tau$-closed in $X_1$. This definition can be considered as a version of *model companions* from model theory. However this version is much weaker because *any $G$-orbit has a $G_\delta$-companion.* Moreover *for any $G_\delta$ $G$-orbit $X_1$ the topologies $\tau$ and $t$ coincide on $X_1$.*

Since orbits of $G$-spaces can be very complicated, it makes sense to study their $G_\delta$-companions. It turns out that in some cases $G_\delta$-companions can by described by methods generalizing model-theoretic forcing.

We illustrate our research by a description of companions of conjugacy classes of $A(\mathbf{Q})$, the group of order-preserving permutations of $\mathbf{Q}$.

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Larisa Maksimova

**Interpolation and Projective Beth Property in Extensions of Minimal Logic**

We consider a family $E(\mathrm{J})$ of logics containing Johansson's minimal logic J. A logic is *consistent* if it is different from the set of all formulas. Denote Neg = J + $\bot$, Int = J + $(\bot \to p)$, Cl= Int + $p \vee (p \to \bot)$.
Let $\mathbf{p}$, $\mathbf{q}$, $\mathbf{q}'$, $\mathbf{r}$ be disjoint lists of variables not containing $y$ and $z$. A logic $L$ has *projective Beth's property PBP* if $L \vdash A(\mathbf{p}, \mathbf{q}, y) \& A(\mathbf{p}, \mathbf{q}', z) \to (y \leftrightarrow z)$ implies $L \vdash A(\mathbf{p}, \mathbf{q}, y) \to (y \leftrightarrow C(\mathbf{p}))$ for some formula C($\mathbf{p}$). $L$ has CIP if $L \vdash A(\mathbf{p}, \mathbf{q}) \to B(\mathbf{p}, \mathbf{r})$ implies that there is $C(\mathbf{p})$ such that $L \vdash A(\mathbf{p}, \mathbf{q}) \to C(\mathbf{p})$ and $L \vdash C(\mathbf{p}) \to B(\mathbf{p}, \mathbf{r})$.
The logic J is characterized by J-algebras $\mathbf{A} = (|\mathbf{A}|; \&, \vee, \to, \bot)$; J-algebra is a Heyting algebra if $\bot$ is the least element of $\mathbf{A}$, and a negative algebra if $\bot$ is the greatest element of $\mathbf{A}$. If $\mathbf{A}$ is a negative algebra and $\mathbf{B}$ a Heyting algebra, denote by $\mathbf{A} \uparrow \mathbf{B}$ a J-algebra with an universe $|\mathbf{A}| \cup |\mathbf{B}|$, where $a \leqslant \bot \leqslant b$ for all $a \in \mathbf{A}$ and $b \in \mathbf{B}$. For $L_1 \in E(\mathrm{Neg})$ and $L_2 \in E(\mathrm{Int})$, denote by $L_1 \uparrow L_2$ a logic characterized by all algebras $\mathbf{A} \uparrow \mathbf{B}$, where $\mathbf{A} \models L_1$ and $\mathbf{B} \models L_2$.

**Theorem 1**. Let $L_1$ and $L_2$ be consistent extensions of Neg and Int respectively. Then
(a) $L_1 \uparrow L_2$ has CIP iff both $L_1$ and $L_2$ have CIP;
(b) $L_1 \uparrow L_2$ has PBP iff $L_1$ has CIP and $L_2$ has PBP.

In [1-3] we described all logics in $E(\mathrm{Int})$ and $E(\mathrm{Neg})$ possessing CIP or PBP. There are exactly 3 consistent extensions of Neg with CIP, 7 consistent extension of Int with CIP and 15 with PBP. It is easy to calculate the number of logics of the form $L_1 \uparrow L_2$ in $E(\mathrm{J})$ which have CIP or PBP. We prove that J + $(\bot \to p) \vee (p \to \bot)$ = Neg$\uparrow$Int and J + $(p \vee (p \to \bot))$ = Neg$\uparrow$Cl.

**Corollary**. The logics J + $(\bot \to p) \vee (p \to \bot)$ and J + $(p \vee (p \to \bot))$ have CIP.

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VETA MURZINA

## The modal logic based on strictly ordered $\alpha$-spaces

In [2] an axiomatization of temporal logics associated with strictly ordered $f$-spaces was obtained. Now the polymodal logic based on $\alpha$-spaces is considered. The concept of $\alpha$-space was introduced by Ershov [1]. The notion of base subsets is of great importance in this definition. We consider a class of linear ordered $\alpha$-spaces with chosen base subset. We prove that the quadruple $\langle X,\ X_0,\ \leqslant,\ \prec\rangle$, where $\leqslant,\ \prec$ are binary relations on $X$ and $X_0 \subseteq X$, is defined by a linear ordered $\alpha$-space if and only if some special conditions are possessed.
Frames $\langle X,\ X_0,\ <,\ \prec\rangle$ are considered. If $\langle X,\ X_0,\ \leqslant,\ \prec\rangle$ is a linear ordered $\alpha$-space with the base subset $X_0$ then $\langle X,\ X_0,\ <,\ \prec\rangle$, where for any $x,\ y \in X$ ($x < y \iff x \leqslant y$ and $x \neq y$), is called a strictly ordered $\alpha$-frame. The modalities $\Box_<$, $\Box_\prec$ associated with $<$ and $\prec$ respectively are introduced. The set $X_0$ is represented as a constant $\beta$. We define a calculus $L\alpha$ by adding the following axioms to the minimal modal calculus $K$:

(I) 1. $\Box_<(\Box_< A_1 \longrightarrow A_2) \vee \Box_<(A_2 \& \Box_< A_2 \longrightarrow A_1)$; 4. $\Box_\prec A \longrightarrow \Box_\prec \Box_< A$;
2. $\Box_< A \longrightarrow \Box_< \Box_< A$; 5. $\Box_\prec A \longrightarrow \Box_< \Box_\prec A$.
3. $\Box_< A \& A \longrightarrow \Box_\prec A$;

(II) 1. $\Diamond_\prec(\beta \& A) \longrightarrow \Diamond_\prec(\beta \& \Diamond_\prec(\beta \& A))$; 3. $\Diamond_< A \longrightarrow (\Diamond_<(\beta \& \Diamond_\prec A)$.
2. $\Diamond_\prec A \longrightarrow (\Diamond_<(\beta \& \Diamond_\prec A) \vee \beta)$;

**Completeness theorem** The calculus $L\alpha$ is complete with respect to the class of all strictly ordered $\alpha$-frames.
Also we prove that $L\alpha$ has the finite model property.

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Wim Ruitenburg

## More Intuitionistic Theories with Quantifier Elimination

We present methods by which to construct new intuitionistic theories admitting quantifier elimination, from known ones. The methods involve the construction of special Kripke models for these new theories, from Kripke models of the known ones.

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DENIS I. SAVELIEV

**Order properties of $\kappa$-dense, Borel, and other sets of reals**

Interrelations between order and descriptive properties of sets of reals are studied.
Let $\kappa$ be a cardinal. A linearly ordered set is called *$\kappa$-dense* iff it has not end-points and each its nontrivial segment has the cardinality $\kappa$. Classical Cantor's theorem sais that all $\aleph_0$-dense sets are order-isomorphic. I consider two possibilities of an extention of this theorem to other sets of reals.
The principle "All $\kappa$-dense sets of reals are order-isomorphic" with an uncountable $\kappa$ was considered for the first time by Baumgartner; he proved its consistency (respectively to ZFC) for $\kappa = \aleph_1$. Shelah proved this for $\kappa = \aleph_2$ (unpublished?); one supposes that the extention to other cardinals is a hard task. I show that the set of all $\kappa$ for which this principle holds is $\omega$-complete:

**Theorem.** *The set of all $\kappa$ such that all $\kappa$-dense sets of reals are order-isomorphic contains the limits of any countable sequence of own elements.*

**Corollary.** *The first cardinal which disproves this principle cannot have countable cofinality.*

Other possibility is to extend Cantor's theorem to Borel (or even determined) sets. In this situation, it is reasonable to require that order-isomorphic sets shall be "sufficiently homogenuous": all nontrivial segments must have one and the same Borel type and category. I show that such principle holds at least for first Borel classes:

**Theorem.** *All $2^{\aleph_0}$-dense meager $F_\sigma$-sets are order-isomorphic.*

Both results are obtained by the same argument, as applications of a certain "abstract" (not concerning real line) theorem about isomorphisms of "self-similar" meager sets.
A connection between results of such kind and small cardinals exists.

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Levan Uridia

**Lattice of varieties of weak monadic algebras**

A pair $(B, d)$ is a weak monadic algebra if $B$ is a Boolean algebra and $d$ is a unary operator satisfying: 1) $d(0) = 0$, 2) $d(p \vee q) = d(p) \vee d(q)$, 3) $dd(p) \leqslant d(p) \vee p$, 4) $p \wedge d(-d(p)) = 0$. It must be pointed out that monadic algebras [1] are exactly those weak monadic algebras satisfying the condition $p \leqslant d(p)$.

We investigate the variety $wMA$ of weak monadic algebras. We show that $wMA$ is discriminator variety and also present a characterization of subdirectly irreducible algebras. Moreover we prove that the lattice of subvarieties of weak monadic algebras $Lat(wMA)$ is isomorphic to the lattice of decreasing sets of all pairs of natural numbers with special ordering. We also give the full description of cyclic weak monadic algebras and show that the number of elements of free cyclic weak monadic algebra is equal to $2^{12}$.

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Guohua Wu

## 1-Generic Splittings of Computably Enumerable Degrees

Say a set $G \subseteq \omega$ is 1-generic if for any $e \in \omega$, there is a string $\sigma \subset G$ such that $\{e\}^{\sigma}(e) \downarrow$ or $\forall \tau \supseteq \sigma(\{e\}^{\tau}(e) \uparrow)$. It is known that $\mathbf{0}'$ can be split into two 1-generic degrees. In this paper, we generalize this and prove that any nonzero computably enumerable degree can be split into two 1-generic degrees. As a corollary, no two computably enumerable degrees bound the same class of 1-generic degrees.

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V. K. Zakharov

## Descriptions of all Tarski sets and all standard supertransitive models of ZF

joint work with E. I. Bunina

Consider the *cumulative collection of sets* $V_\alpha$ such that $V_0 \equiv \emptyset$, $V_{\alpha+1} = V_\alpha \cup \mathcal{P}(V_\alpha)$, $V_\alpha = \cup[V_\beta | \beta \in \alpha]$ for limit ordinals $\alpha$.
Zermelo (1930) and Shepherdson (1951) proved that a set $U$ is a supertransitive standard model of NBG iff $U = V_{\varkappa+1}$ for some (strongly) inaccessible cardinal $\varkappa$, where $U$ is called *supertransitive* iff $\forall x \in U \forall y((y \in x \Rightarrow y \in U) \wedge (y \subset x \Rightarrow y \in U))$.
Tarsky (1938) introduced the notion of *a Tarsky set* $U$ such that $x \in U \Rightarrow x \subset U$, $x \in U \Rightarrow \mathcal{P}(x) \in U$, $(x \subset U \wedge \forall f(f \in U^x \Rightarrow rng\, f \neq U) \Rightarrow x \in U$.
Tarsky (1938) proved that the set $V_\varkappa$ for every inaccessible cardinal $\varkappa$ is a Tarsky set, but the converse was not proved. We prove that *for a set $U$ the following assertions are equivalent*:
1) *$U = V_\varkappa$ for some inaccessible cardinal $\varkappa$*;
2) *$\mathcal{P}(U)$ is a supertransitive standard model of* NBG;
3) *$U$ is an uncountable Tarsky set.*
Montague and Vaught (1959) proved that for every inaccessible cardinal $\varkappa$ there exists an ordinal $\theta < \varkappa$, such that $\theta$ is not inaccessible and $V_\theta$ is a supertransitive standard model of ZF.
Every formula $\varphi(x, y; \vec{p})$ for every set $A$ assigns the *scheme correspondence* $[\varphi(x, y; \vec{p})|A] \equiv \{z \in A * A | \exists x, y \in A(z = \langle x, y\rangle \wedge \varphi^A(x, y; \vec{p}))\}$.
An ordinal $\varkappa$ will be called *scheme-regular* if $\forall \vec{p} \in V_\varkappa \forall \alpha(\alpha \in \varkappa \wedge ([\varphi(x, y; \vec{p})|V_\varkappa] : \alpha \to \varkappa \Rightarrow \cup rng\, [\varphi(x, y; \vec{p})|V_\varkappa] \in \varkappa)$ for any formula $\varphi$ of ZF.
An ordinal $\varkappa > \omega$ will be called *scheme-inaccessible* if $\varkappa$ is scheme-regular and $\forall \alpha(\alpha \in \varkappa \Rightarrow |\mathcal{P}(\alpha)| \in \varkappa)$.
We prove that *for a set $U$ the following assertions are equivalent*:
1) *$U = V_\varkappa$ for some scheme-inaccessible cardinal $\varkappa$*;
2) *$U$ is a supertransitive standard model of* ZF.

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